



UDC 517

On Necessary Conditions for a Minimum of a Quadratic Functional with a Stieltjes Integral and Zero Coefficient of the Highest Derivative on the Part of the Interval

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In this paper we obtain a necessary condition for an extremum of a quadratic functional with a Stieltjes integral in the case where the coefficient of the highest derivative may vanish on a part of the interval. It is shown that the resulting mathematical model has the property of non-degeneracy. It is proved that a Variable boundary problem that arises as a necessary condition for an extremum is an «intermediate» position between the boundary value problems of fourth- and second-order — the solution space has dimension three.

Key words: functional, a necessary condition, Stieltjes integral, derivative on the measure.

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Matrix Representation of Dilation Operator on the Product of Zero-Dimensional Locally Compact Abelian Groups

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In the real wavelet analysis d -dimensional dilation operator may be written with the help of an integer-valued $d \times d$ matrix. We find the matrix representation of the dilation operator on the product of zero-dimensional locally compact Abelian groups.

Key words: zero-dimensional group, delation operator, multiresolution analysis.

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On 2-fold Completeness of the Eigenfunctions for the Strongly Irregular Quadratic Pencil of Differential Operators of Second Order

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A class of strongly irregular pencils of ordinary differential operators of second order with constant coefficients is considered. The roots of the characteristic equation of the pencils from this class are supposed to lie on a straight line coming through the origin and on the both side of the origin. Exact interval on which the system of eigenfunctions is 2-fold complete in the space of square summable functions is found.

Key words: quadratic pencil, second order pencil, pencil of ordinary differential operators, two-point boundary conditions, homogeneous differential expression with constant coefficients, completeness of the system of eigenfunctions, non-completeness of the system of eigenfunctions.

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Structure of the Inverse for the Integral Operator of Special Kind

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Algebra (with identity) generated by integral operators on the spaces of continuous periodic functions is considered. This algebra is proved to be an inverse-closed subalgebra in the algebra of all bounded linear operators.

Key words: Banach space, integral operator, Bochner–Phillips theorem, Fourier series of an operator, inverse-closedness of a subalgebra, Wiener’s pair of algebras.

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The Problem of Leont'ev on Entire Functions of Completely Regular Growth

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We consider an entire function of exponential type with all its zeros are simple and form a sequence with the index condensation zero. On the set of zeros a function of its derivative is growing quickly. Required to determine whether original function have complete regularity of growth. This problem, which arose in the theory of representation of analytic functions by exponential series was posed by A. F. Leontiev more than forty years ago and has not yet been solved. In this paper we show that the aforesaid problem a positive solution if the function is «not too small» on a straight line.

Key words: Leont'ev problem, function of completely regular growth, index of condensation.

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A U -set for System of Character of the Zero-dimensional Group under Convergent over Cubes

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In this work we consider system of characters of the compact zero-dimensional group G and study uniqueness sets for N -fold multiple series for system of character a zero-dimensional group under convergent over cubes (in other words, U -sets). We prove that every finite set is a U -set and show that countable set with only one limit point is a U -set.

Key words: compact zero-dimensional group, U -set, N -fold multiple series under convergent over cubes.

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UDC 539.3

The Equilibrium Equations of Shells in the Coordinates of the General Form

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A mathematical model of homogeneous elastic shells is considered under kinematics Reissner–Mindlin type. Through direct (coordinateless) methods of the tensor calculus equations of equilibrium are obtained in terms of displacements in an arbitrary (not necessarily orthogonal) coordinate system, taking into account the asymmetry of the location of the front surface. For a spherical shells proposed procedure for constructing solutions, based on the method of spectral decomposition, which describes the stress-strain state at the potential power and torque static loads.

Key words: sphere shell, equilibrium equations, analytical solutions, spectral decomposition, eigenfunctions.

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Modeling of the Shock System Motion with Impacts about Hard Barriers

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We have developed a model of a shock system with a resilient member under periodic force action including impacts about hard barriers. In order to model the shock system we have developed a program providing a computational solution for differential equations of a subject motion taking into account conditions of periodicity and collision, graphical and numerical reproduction of motion parameters in the simulation process. We have performed simulation of modes of the shock system. In the process of computational experiments parameters of the system response have been estimated and corrected upon the results.

Key words: modeling, periodic impact, model of the impact, shock system, motion with impacts about barriers, periodic motion mode, computational experiment.

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Covariant Field Equations and d -tensors of Hyperbolic Thermoelastic Continuum with Fine Microstructure

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A non-linear mathematical model of hyperbolic thermoelastic continuum with fine microstructure is proposed. The model is described in terms of 4-covariant field theoretical formalism. Fine microstructure is represented by d -tensors, playing role of extra field variables. A Lagrangian density for hyperbolic thermoelastic continuum with fine microstructure is given and the corresponding least action principle is formulated. 4-covariant field equations of hyperbolic thermoelasticity are obtained. Constitutive equations of microstructural hyperbolic thermoelasticity are discussed. Virtual microstructural inertia is added to the considered action density. It is also concerned to the thermal inertia. Variational symmetries of the thermoelastic action are used to formulate covariant conservation laws in a plane space–time.

Key words: thermoelasticity, microstructure, field, extra field, action, Lagrangian, covariance, symmetry, conservation law, d -tensor, 4-current, energy–momentum tensor.

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Investigation of Surface Roughness at Micro-scale and Mechanical Response in the Contemporary Bio-polymer Sutures by the Nanoindentation

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An investigation of properties of contemporary suture materials (surgical threads) is the state-of-art challenge in biomechanics. To improve an effectiveness of sutures application, an analysis of structure and elastic properties by the atomic force microscopy and scanning electron microscopy is necessary to be performed. As a result, the force-indentation depth dependences were plotted to obtain the Young's modulus of the thread at micro-scale taking into account influence of indentation area localization; moreover, the thread surface roughness was evaluated at an area of 5×5 and 10×10 micrometers.

Key words: atomic force microscopy (AFM), biopolymer, suture material, nanoindentation, Hurst exponent.

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Optimal Filtration of Matrix Gaussian Random Processes in Planes Lateral Motion Problem

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In practice, observation problem is more complex because of random influences (noises): wind effects plane course, sensor errors distort object position view. In order to reduce noise filters are used. Proposed to carry out a simultaneous filtering of identical objects motion by defining problem in matrix variables. To achieve physical realizability controlled matrix filter was proposed. Statements that allow to find the optimal solution was proved.

Key words: matrix filtration, n -covariance matrix, square-law functional.

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Using Parallel Computing Technologies for Modeling of Metallic Photonic Crystals

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This article presents opportunities of using parallel computing technologies Message Passing Interface and Open Computing Language for modeling of metallic photonic crystals with the method of Green's functions and integral equations. The efficiency of these technologies is analyzed and the results are presented.

Key words: parallel computing, MPI, OpenCL, photonic crystals.

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Ordered Automata and Tolerant Images of FDA

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Finite deterministic automaton (FDA) with partially ordered (an ordered automaton) sets of states, input and output symbols is described in the article. The mapping of FDA on an ordered automaton, which is named « p -morphism» is defined. It is shown that so called tolerant images, which are constructed with the help of compatible tolerances on the set of states of FDA, are particular case of ordered automata, which are connected with the original automaton by a p -morphism. Necessary and sufficient conditions are defined, under which an ordered automaton is a tolerant image of the original one.

Key words: finite deterministic automaton, tolerant image, ordered automaton, compatible tolerance, covering, partial order.

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T-irreducible Extension for Union of Paths and Cycles

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A graph H with $n + 1$ nodes is an extension of a graph G with n nodes if each maximal subgraph of H contains G . Trivial extension of a graph G is the connection of graph G and the singleton graph (i.e. we add one node to the graph G and this node join with each node of G). T-irreducible extension of graph G is an extension of the graph G which is obtained by removing maximal set of edges from the trivial extension of G . One of T-irreducible extensions is constructed for an arbitrary union of cycles and paths.

Key words: graph, T-irreducible extensions, union of paths and cycles.

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On Upper Bound of Vertex Distinguishing Word Length on Vertex Labeled Graph

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The problem of vertex distinguishing on vertex labeled graphs is considered. Two vertices are called distinguishable if associated languages over the alphabet of labels are different. A linear upper bound of vertex distinguishing word length equal to half the number of vertices is obtained.

Key words: vertex labeled graphs, languages over the label alphabet, vertex equivalence.

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Analysis of Closed Unreliable Queueing Networks with Batch Movements of Customers

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Closed unreliable queueing network with batch movements is considered. The main result of the paper is the steady state distribution for given type queueing networks.

Key words: queueing networks, unreliable server, batch movements of customers, analysis of queueing networks.

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