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Martingale Inequalities in Symmetric Spaces with Semimultiplicative Weight

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Let (Ω, Σ, P) be a complete probability space, $\mathscr{F} = \{\mathscr{F}_n\}_{n=0}^{\infty}$ be an increasing sequence of σ-algebras such that $\cup_{n=0}^{\infty}\mathscr{F}_n$ generates Σ. If $f = \{f_n\}_{n=0}^{\infty}$ is a martingale with respect to $\mathscr{F}$ and $\mathbb{E}_n$ is the conditional expectation with respect to $\mathscr{F}_n$, then one can introduce a maximal function $M(f) = \sup_{n\geqslant 0}|f_n|$ and a square function $S(f) = \left(\sum\limits_{i=0}^{\infty}|f_i - f_{i-1}|^2\right)^{1/2}$, $f_{-1} = 0$. In the case of uniformly integrable martingales there exists $g \in L^1(\Omega)$ such that $\mathbb{E}_n g = f_n$ and we consider a sharp maximal function $f^\sharp = \sup_{n\geqslant 0}\mathbb{E}_n|g - f_{n-1}|$. The result of Burkholder – Davis – Gundy is that $C_1\|M(f)\|_p \leqslant \|S(f)\|_p \leqslant C_2\|M(f)\|$ for $1 < p < \infty$, where $\|\cdot\|_p$ is the norm in $L^p(\Omega)$ and $C_2 > C_1 > 0$. We call the inequality of type $\|M(f)\|_p \leqslant C\|f^\sharp\|_p$, $1 < p < \infty$ Fefferman – Stein inequality. It is known that Burkholder – Davis – Gundy martingale inequality is valid in rearrangement invariant Banach function spaces with non-trivial Boyd indices. We prove this inequality in a more wide class of symmetric spaces (the last notion is defined as in the famous monograph by S. G. Krein, Yu. I. Petunin and E. M. Semenov) with semimultiplicative weight. Also, the Fefferman – Stein type inequalities of sharp maximal function and sharp square functions are obtained in this class of symmetric spaces.

Keywords: martingale, maximal function, maximal sharp function, square function of martingale, Burkholder – Davis – Gundy inequality, semimultiplicative function.

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