Multiple Completeness of the Root Functions of the Pencils of Differential Operators with Constant Coefficients and Splitting Boundary Conditions

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In the space of square summable functions on the main segment [0,1], the class of polynomial pencils of ordinary differential operators of the $n$-th order is considered. The coefficients of the differential expression are assumed to be constants. The boundary conditions are assumed to be splitting and two-point at the ends 0 and 1 ($l$ of boundary conditions is taken only at the point 0, and the remaining $n - l$ is taken at the point 1). The differential expression and the boundary forms are assumed to be homogeneous, that is, they contain only main parts. It is supposed that roots of the characteristic equation of the pencils of this class are simple, non-zero and lie on two rays emanating from the origin in quantities $k$ and $n - k$. Sufficient conditions of $m$-fold system completeness of root functions of the pencils of this class in the space of square integrable functions on the main segment (with a possible finite defect) are formulated. The multiplicity $m$ completeness depends on the relations of the parameters $n$, $l$ and $k$. In this case, it is assumed that some completely concrete determinants differ from zero. These determinants are constructed from the coefficients of the boundary forms of the pencil and the roots of the characteristic equation. An upper bound on a possible finite defect is given.

Keywords: pencil of ordinary differential operators, polynomial pencil of differential operators, homogeneous differential expression, homogeneous boundary forms, multiple completeness, root functions, eigen- and associated functions, derived chains, splitting boundary conditions.

Received: 07.04.2018 / Accepted: 05.04.2019 / Published online: 28.05.2019

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Cite this article as: