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On the Geometric Structure of the Continuos Mappings Preserving the Orientation of Simplexes

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It is easy to show that if a continuous open map preserves the orientation of all simplexes, then it is affine. The class of continuous open maps $f : D \subset \mathbb{R}^m \to \mathbb{R}^n$ that preserve the orientation of simplexes from a given subset of a set of simplexes with vertices in the domain $D \subset \mathbb{R}^m$ is considered. In this paper, questions of the geometric structure of linear inverse images of such mappings are studied. This research is based on the key property proved in the article: if a map preserves the orientation of simplexes from some subset B of the set of all simplexes with vertices in the domain D, then the inverse image of the hyperplane under such a mapping can not contain the vertices of a simplex from B. Based on the analysis of the structure of a set possessing this property, one can obtain results on its geometric structure. In particular, the paper proves that if a continuous open map preserves the orientation of a sufficiently wide class of simplexes, then it is affine. For some special classes of triangles in $\mathbb{R}^2$ with a given condition on its maximal angle it is shown that the inverse image of a line is locally a graph (in some case a Lipschitzian) of a function in a suitable Cartesian coordinate system.

Key words: simplex, orientation of simplex,continuous mapping, monotone function.

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