Embeddings of Generalized Bounded Variation Function Spaces into Spaces of Functions with Given Majorant of Average Modulus of Continuity

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In the present paper we study embeddings of some spaces of functions of generalized bounded variation into classes of functions with given majorant of average modulus of continuity introduced by B. Sendov and V. Popov. We consider the spaces $\Lambda BV^{(p)}$ of functions of bounded $(\Lambda - p)$-variation suggested by D. Waterman (for $p = 1$) and M. Shiba (for $p > 1$) and spaces $V(v(n))$ of functions with given majorant of its modulus of variation. The last quantity was introduced by Z. A. Chanturia. The necessary and sufficient conditions of such embeddings are proved. Earlier similar embeddings into classes with given majorant of usual integral modulus of continuity were studied by Yu. E. Kuprikov, U. Goginava and V. Tskhadaia, M. Hormozi et al. Applications of obtained results to estimates of errors for some quadrature rules are given.

Key words: average modulus of continuity, $\Lambda BV^{(p)}$ space, modulus of variation, embedding, quadrature rule.

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