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Linear Difference Equation of Second Order in a Banach Space and Operators Splitting

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In differential and difference equations classical textbooks, the n-th order differential and difference equations reducing by standard substitution to first-order differential and difference equations system is described. Each of the cohering equations can be written in the operator form. Naturally there is a question of coincidence of a number of properties of differential and difference equations (operators) of the second order and the corresponding functional equations (operators) of first order. In this paper we study the second order linear difference equation in the complex Banach space with bounded operator coefficients. The first theorem establishes the simultaneous invertibility of the second-order difference operator and the corresponding first-order difference operator, and the inverse operator formula is given. The research is conducted under conditions of the corresponding „algebraic" operator equation with separated roots. Theorem 2 establishes the second-order operator matrix and block-diagonal operator matrix similarity. In pair of operator roots separation condition in Theorem 3, the necessary and sufficient condition for the second and the first order difference operators invertibility is obtained. In Theorem 4 we obtain the operators under consideration inverse operators formalism (formula). In Theorems 5 and 6 for bounded solutions on the set of non-negative integers an asymptotic formalism of these solutions is obtained using operator-valued functions, this formalism can be called splitting at infinity.

Key words: Banach space, difference equation of second order, operators splitting.

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