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Approximation of Control for Singularly Perturbed System with Delay with Integral Quadratic Constraints

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The purpose of the work is the development and theoretical substantiation of analytical approximate or asymptotic methods for solving optimal control problems for singularly perturbed systems with constant delay in phase variables under conditions of uncertainty with respect to the initial data. For achievement of a goal the control problem for the singularly perturbed system with delay with indeterminate initial conditions and integral quadratic constraints on the control resources according to the minimax criterion is considered. A limit problem is formulated for which the quality functional is chosen in a special way. The proposed method is based on the idea of separating the asymptotics of the ensemble of trajectories of a singularly perturbed system with delay and representing the fundamental matrix of solutions divided into blocks in accordance with the dimensions of fast and slow variables in the form of a uniformly convergent sequence. We propose a procedure to construct an initial approximation of control response for the minimax problem of control. The work uses problem statements, concepts, methods and results of control theory under uncertainty, as well as methods of the theory of extremal problems, asymptotic analysis methods, classical methods of convex and real analysis.

Key words: singularly perturbed system with delay, optimal control, fundamental matrix.

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