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Connections of Nonzero Curvature on Three-dimensional Non-reductive Spaces

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When a homogeneous space admits an invariant affine connection? If there exists at least one invariant connection then the space is isotropy-faithful, but the isotropy-faithfulness is not sufficient for the space in order to have invariant connections. If a homogeneous space is reductive, then the space admits an invariant connection. The purpose of the work is a description of three-dimensional non-reductive homogeneous spaces, admitting invariant affine connections of nonzero curvature only, and the affine connections, curvature and torsion tensors. The basic notions, such as an isotropically-faithful pair, an affine connection, curvature and torsion tensors, a reductive space are defined. The local description of three-dimensional non-reductive homogeneous spaces, admitting connections of nonzero curvature only, is given. The local classification of such spaces is equivalent to the description of the effective pairs of Lie algebras. All invariant affine connections on those spaces are described, curvature and torsion tensors are found.

Key words: affine connection, curvature tensor, reductive space, transformation group, Lie algebra.

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