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Adjustment of Functions and Lagrange Interpolation Based on the Nodes Close to the Legendre Nodes

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It is well known that the Lagrange interpolation of a continuous function based on the Chebyshev nodes may be divergent everywhere (for arbitrary nodes, almost everywhere) like the Fourier series of a summable function. On the other hand any measurable almost everywhere finite function can be "adjusted" in a set of arbitrarily small measure such that its Fourier series will be uniformly convergent. The question arises: does the class of continuous functions have a similar property with respect to any interpolation process? In the present paper we prove that there exists a matrix of nodes $\mathfrak{M}_\gamma$ arbitrarily close to the Legendre matrix with the following property: any function $f \in C[-1, 1]$ can be adjusted in a set of arbitrarily small measure such that the interpolation process of adjusted continuous function g based on the nodes $\mathfrak{M}_\gamma$ will be uniformly convergent to g on $[a, b] \subset (-1, 1)$.

Key words: Lagrange interpolation, Legendre orthogonal polynomials, adjustment of functions.

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