UDC 517.587

Polynomials Orthogonal with Respect to Sobolev Type Inner Product Generated by Charlier Polynomials

I. I. Sharapudinov, I. G. Guseinov

Idris I. Sharapudinov, https://orcid.org/0000-0002-2290-9878, Dagestan Scientific Center of RAS, 45, M. Gadzhieva Str., Makhachkala, 367025, Russia, sharapud@mail.ru

Ibraghim G. Guseinov, https://orcid.org/0000-0002-3888-6383, Dagestan State University, 43-a, M. Gadzhieva Str., Makhachkala, 367000, Russia; Dagestan Scientific Center RAS, 45, M. Gadzhieva Str., Makhachkala, 367025, Russia, ibraa2g@gmail.ru

The problem of constructing of the Sobolev orthogonal polynomials $s_{r,n}^{\alpha}(x)$ generated by Charlier polynomials $s_n^{\alpha}(x)$ is considered. It is shown that the system of polynomials $s_{r,n}^{\alpha}(x)$ generated by Charlier polynomials is complete in the space $W^r_{l_\rho}$, consisted of the discrete functions, given on the grid $\Omega=\{0,1,\ldots\}$. $W^r_{l_\rho}$ is a Hilbert space with the inner product $\langle f,g\rangle$. An explicit formula in the form of $s_{r,k+r}^{\alpha}(x)=\sum\limits_{l=0}^{k} b_l^r x^{[l+r]}$, where $x^{[m]}=x(x-1)\ldots(x-m+1)$, is found. The connection between the polynomials $s_{r,n}^{\alpha}(x)$ and the classical Charlier polynomials $s_n^{\alpha}(x)$ in the form of $s_{r,k+r}^{\alpha}(x)=U_k^r\left[s_{k+r}^{\alpha}(x)-\sum\limits_{\nu=0}^{r-1}V_{k,\nu}^r x^{[\nu]}\right]$, where for the numbers U_k^r, $V_{k,\nu}^r$ we found the explicit expressions, is established.

Key words: Sobolev orthogonal polynomials, Charlier polynomials, Sobolev-type inner product.

DOI: 10.18500/1816-9791-2018-18-2-196-205

References

1. Iserles A., Koch P. E., Norsett S. P., Sanz-Serna J. M. On polynomials orthogonal with respect to certain Sobolev inner products. *J. Approx. Theory*, 1991, vol. 65, iss. 2, pp. 151–175. DOI: 10.1016/0021-9045(91)90100-O
2. Marcellan F., Alfaro M., Rezola M. L. Orthogonal polynomials on Sobolev spaces: old and new directions. *J. Comput. Appl. Math.*, 1993, vol. 48, iss. 1–2, pp. 113–131. DOI: 10.1016/0377-0427(93)90318-6
3. Meijer H. G. Laguerre polynomials generalized to a certain discrete Sobolev inner product space. *J. Approx. Theory*, 1993, vol. 73, iss. 1, pp. 1–16. DOI: 10.1006/jath.1993.1029
4. Kwon K. H., Littlejohn L. L. The orthogonality of the Laguerre polynomials $\{L_n^{(-k)}(x)\}$ for positive integers k. *Ann. Numer. Anal.*, 1995, vol. 2, pp. 289–303.
5. Kwon K. H., Littlejohn L. L. Sobolev orthogonal polynomials and second-order differential equations. *Rocky Mountain J. Math.*, 1998, vol. 28, pp. 547–594. DOI: 10.1216/rmjm/1181071786
6. Marcellan F., Xu Y. On Sobolev orthogonal polynomials. *arXiv:1403.6249v1* [math.CA]. 25 Mar 2014. 40 p.
7. Sharapudinov I. I., Gadzhieva Z. D. Sobolev orthogonal polynomials generated by Meixner polynomials. *Izv. Saratov Univ. (N.S.), Ser. Math. Mech. Inform.*, 2016, vol. 16, iss. 3, pp. 310–321 (in Russian). DOI: 10.18500/1816-9791-2016-16-3-310-321
8. Sharapudinov I. I. *Smeshannyj rjady po ortogonal'nym polinomam* [Mixed Series in Orthogonal Polynomials]. Makhachkala, Izd-vo DNC RAN, 2004. 176 p. (in Russian).
9. Sharapudinov I. I. *Mnogochleny, ortogonal'nye na setkah* [Polynomials Orthogonal on Grids]. Makhachkala, Izd-vo Dag. gos. ped. un-ta, 1997. 252 p. (in Russian).

10. Bateman H., Erdelyi A. *Higher Transcendental Functions*. Vol. 2. New York, McGraw-Hill Book Company, 1953. 396 p. (Rus. ed.: Moscow, Nauka, 1974. 296 p.)
11. Shirjaev A. N. *Verojatnost'-1* [Probability-1]. Moscow, MTsNMO, 2007. 552 p. (in Russian).

Cite this article as:
Sharapudinov I. I., Guseinov I. G. Polynomials Orthogonal with Respect to Sobolev Type Inner Product Generated by Charlier Polynomials. *Izv. Saratov Univ. (N. S.), Ser. Math. Mech. Inform.*, 2018, vol. 18, iss. 2, pp. 196–205 (in Russian). DOI: 10.18500/1816-9791-2018-18-2-196-205.