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Application of Generalized Differential Quadrature Method to Two-dimensional Problems of Mechanics

M. A. Barulina

Marina A. Barulina, https://orcid.org/0000-0003-3867-648X, Institute of Precision Mechanics and Control, Russian Academy of Sciences, 24, Rabochaya Str., Saratov, Russia, 410028, marina@barulina.ru

The application of the generalized differential quadrature method to the solution of two-dimensional problems of solid mechanics is discussed by an example of the sample analysis of vibrations of a rectangular plate under various types of boundary conditions. The differential quadrature method (DQM) is known as an effective method for resolving differential equations, both ordinary and partial. The main problems while using DQM, as well as other quadrature methods, are choosing the distribution for construction of the points grid and determination of the weight coefficients, and incorporarting boundary conditions in the resolving system of linear algebraic equations. In the present study a generalized approach to accounting the boundary conditions is proposed and a universal algorithm for the composition of a resolving algebraic system is given. In the paper it is shown by an example of model analysis of a rectangular plate vibrations that the DQM allows us to effectively resolve two-dimensional problems of solid mechanics gaining an acceptable accuracy with a relatively small number of points on the grid. The latter is provided by the aid of the classical non-uniform Chebyshev – Gauss – Lobatto distribution and generalized approach to accounting of the boundary conditions.

Key words: differential quadrature method, numerical methods, differential equations, eigenfrequencies, rectangular plate.

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Таблица 1 / Table 1

Собственные частоты квадратной пластины при разных граничных условиях
The eigenfrequencies of a square plate with different boundary conditions

Закрепление / Boundary Conditions	Сетка / Grid	Ω_1	Ω_2	Ω_3
Ж-Ж-Ж-Ж / C-C-C-C	10×10	35.9378	73.3186	73.3186
	15×15	36.0018	73.3940	73.3940
	20×20	35.9848	73.4590	73.4590
	АНАЛИТ / ANALYT	35.9920	73.4130	73.4130
СО-СО-СО-СО / SS-SS-SS-SS	10×10	19.7393	49.3208	49.3208
	15×15	19.7392	49.3480	49.3480
	20×20	19.7332	49.3480	49.3480
	АНАЛИТ / ANALYT	19.7392	49.3480	49.3480
Н-Н-Н-Н / F-F-F-F	10×10	13.2810	19.6053	24.2585
	15×15	13.4670	19.5951	24.2581
	20×20	13.4601	19.5966	24.2654
	АНАЛИТ / ANALYT	13.4890	19.5960	24.4320
СО-Ж-СО-Ж / SS-C-SS-C	10×10	28.9490	54.6923	69.1478
	15×15	28.9509	54.7431	69.3271
	20×20	28.9509	54.7431	69.3270
	АНАЛИТ / ANALYT	28.9509	54.7431	69.3270

Note. F — free edge; SS — simply supported edge; C — clamped edge; ANALYT — analytical values.

Таблица 2 / Table 2

Значения первой частоты Ω_1 прямоугольной пластины при разных граничных условиях и различных соотношениях сторон λ

The values of the first frequency Ω_1 of a rectangular plate with different boundary conditions and different aspect ratios λ

Закрепление / Boundary Conditions	Решатель / Method	$\lambda = 2/5$	$\lambda = 2/3$	$\lambda = 1$	$\lambda = 3/2$	$\lambda = 5/2$
Ж-Ж-Ж-Ж / C-C-C-C	ОМДК / GDQM	23.644	27.099	36.0018	60.7723	147.775
	АНАЛИТ / ANALYT	23.648	27.010	35.9920	60.772	147.80
Ж-Ж-Ж-СО / C-C-C-SS	ОМДК / GDQM	23.4390	25.8607	31.8309	48.163	107.0433
	АНАЛИТ / ANALYT	23.440	25.861	31.829	48.167	107.07
Ж-Ж-СО-СО / C-C-SS-SS	ОМДК / GDQM	19.8475	19.9516	27.0555	44.8912	105.2972
	АНАЛИТ / ANALYT	16.849	19.952	27.056	44.893	105.31
Ж-Ж-СО-Н / C-C-SS-F	ОМДК / GDQM	15.5818	15.8680	16.7833	21.0635	32.6753
	АНАЛИТ / ANALYT	15.696	16.287	17.615	21.035	33.578
Ж-СО-СО-Н / C-SS-SS-F	ОМДК / GDQM	15.5424	15.6779	16.1528	18.9015	22.1081
	АНАЛИТ / ANALYT	15.649	16.067	16.865	18.540	23.067
Ж-Н-Н-Н / C-F-F-F	ОМДК / GDQM	3.5075	3.5109	3.5434	3.2364	3.5505
	АНАЛИТ / ANALYT	3.5107	3.5024	3.4917	3.4772	3.4562
СО-СО-СО-СО / SS-SS-SS-SS	ОМДК / GDQM	11.4487	14.2561	19.7392	32.0762	71.5546
	АНАЛИТ / ANALYT	11.4487	14.2561	19.7392	32.0762	71.5564
Н-Н-Н-Н / F-F-F-F	ОМДК / GDQM	3.4313	8.9317	13.4670	20.0924	21.4454
	АНАЛИТ / ANALYT	3.4629	8.9459	13.4890	20.128	21.643
СО-Ж-СО-Ж / SS-C-SS-C	ОМДК / GDQM	12.1347	17.3730	28.9509	56.3481	145.4839
	АНАЛИТ / ANALYT	12.1347	17.3730	28.9509	56.3481	145.4839

Note. F — free edge; SS — simply supported edge; C — clamped edge; ANALYT — analytical values; GDQM — generalized differential quadrature method.

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