UDC 517.984

Inverse Problem for Sturm – Liouville Operators in the Complex Plane

A. A. Golubkov

Andrey A. Golubkov, https://orcid.org/0000-0002-5265-1310, The Advanced Educational Scientific Center (faculty) "— Kolmogorov's boarding school of Moscow State University, 11, Kremenchugskaya Str., Moscow, 121357, Russia, andrej2501@yandex.ru

The inverse problem for the standard Sturm – Liouville equation with a spectral parameter ρ and a potential function, piecewise-entire on a rectifiable curve $\gamma \subset$ **C**, on which only the starting point is given, is studied for the first time. A function Q that is bounded on a curve γ is piecewise-entire on it if γ can be splitted by a finite number of points into parts on which Q coincides with entire functions, different in neighboring parts. The split points, the initial and final points of the curve are called critical points. The problem is to find all the critical points of the curve γ and the potential on it by the column or row of the transfer matrix $\hat{P}$ along γ. On the basis of the obtained asymptotics of matrix $\hat{P}$ for $|\rho| \to \infty$, it is proved that if at least one of its elements is bounded for $\forall \rho \in$ **C**, then the curve γ degenerates to a point after removing all „invisible loops". An „invisible loop" is a loop of the curve γ (with a given piecewise-entire function) whose knot coincides with two successive critical points. The uniqueness of the solution of the inverse problem for curves without „invisible loops" is proved. On the example of the inverse problem for the equation $\frac{d}{dx}\left(\frac{1}{r(x)}\frac{dy}{dx}\right) + \left(q(x) - r(x)\lambda^2\right) y(x) = 0$ with a piecewise-entire function $q(x)$ and a piecewise constant function $r(x) \neq 0$ on the segment of the real axis, the usefulness of the results obtained in the article is shown for the study of inverse problems for generalized Sturm – Liouville equations, which can be reduced to the type studied in the article.

Key words: Sturm – Liouville equation on a curve, piecewise-entire potential function, transfer matrix, asymptotics, inverse spectral problem.

DOI: 10.18500/1816-9791-2018-18-2-144-156

Acknowledgements: The author is grateful to Vyacheslav A. Yurko and Mikhail Yu. Ignatiev for useful discussions of the results obtained.

References

1. Marchenko V. A. *Sturm – Liouville operators and their applications*. Birkhäuser, 1986. 393 p. (Russ.ed. : Kiev, Naukova Dumka, 1977. 331 p.)
2. Levitan B. M. *Inverse Sturm – Liouville problems*. Utrecht, VNU Sci. Press, 1987. 246 p. (Russ.ed. : Moscow, Nauka, 1984. 240 p.)
3. Yurko V. A. *Method of Spectral Mappings in the Inverse Problem Theory*. Inverse and Ill-posed Problems Series. Utrecht, VSP, 2002. 316 p.
4. Golubkov A. A., Makarov V. A. Inverse spectral problem for a generalized Sturm - – Liouville equation with complex-valued coefficients. *Differential Equations*, 2011, vol. 47, no. 10, pp. 1514–1519. DOI : 10.1134/S0012266111100156
5. Golubkov A. A., Makarov V. A. Reconstruction of the coordinate dependence of the diagonal form of the dielectric permittivity tensor of a one-dimensionally inhomogeneous medium. *Moscow University Physics Bulletin*, 2010, vol. 65, no. 3, pp. 189–194. DOI : 10.3103/S0027134910030070
6. Angeluts A. A., Golubkov A. A., Makarov V. A., Shkurinov A. P. Reconstruction of the spectrum of the relative permittivity of the plane-parallel plate from the angular dependences of its transmission coefficients. *JETP Letters*, 2011, vol. 93, no. 4, pp. 191—194. DOI : 10.1134/S0021364011040047

7. Levitan B. M., Sargsyan I. S. *Sturm – Liouville and Dirac Operators*. Mathematics and Its Applications (Soviet Series), vol. 59. Dordrecht, Springer, 1990. 350 p. (Russ. ed.: Moscow, Nauka, 1988. 432 p.). DOI 10.1007/978-94-011-3748-5
8. Ishkin Kh. K. Necessary Conditions for the Localization of the Spectrum of the Sturm – Liouville Problem on a Curve. *Math. Notes*, 2005, vol. 78, no. 1, pp. 64–75. DOI: 10.1007/s11006-005-0100-5.
9. Fedoryuk M. V. *Asymptotic analysis: linear ordinary differential equations*. Berlin, Springer–Verlag, 1993. 363 p. (Russ. ed.: Moscow, Nauka, 1983. 352 p.)
10. Coddington E. A., Levinson N. *Theory of ordinary differential equations*. New York, McGraw-Hill, 1955. 429 p. (Russ. ed.: Moscow, Izd-vo inostr. lit., 1958. 475 p.)
11. Wasow W. *Asymptotic expansions for ordinary differential equations*. New York, Dover Publications, 1988. 384 p. (Russ. ed.: Moscow, Mir, 1968. 465 p.)
12. Ishkin Kh. K. Localization criterion for the spectrum of the Sturm – Liouville operator on a curve. *St. Petersburg Mathematical Journal*, 2017, vol. 28, no. 1, pp. 37–63. DOI: 10.1090/spmj/1438
13. Ishkin Kh. K. On the uniqueness criterion for solutions of the Sturm – Liouville equation. *Math. Notes*, 2008, vol. 84, no. 4, pp. 515—528. DOI : 10.1134/S000143460809023X
14. Ishkin Kh. K. On a trivial monodromy criterion for the Sturm – Liouville equation. *Math. Notes*, 2013, vol. 94, no. 4, pp. 508–523. DOI : 10.1134/S0001434613090216

Cite this article as:
Golubkov A. A. Inverse Problem for Sturm – Liouville Operators in the Complex Plane. *Izv. Saratov Univ. (N. S.), Ser. Math. Mech. Inform.*, 2018, vol. 18, iss. 2, pp. 144–156 (in Russian). DOI: 10.18500/1816-9791-2018-18-2-144-156.