UDC 517.51

On Binary B-splines of Second Order

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The classical B-spline is defined recursively as the convolution $B_{n+1} = B_n * B_0$, where B_0 is the characteristic function of the unit interval. The classical B-spline is a refinable function and satisfies the Riesz inequality. Therefore any B-spline B_n generates the Riesz multiresolution analysis (MRA). We define binary B-splines, obtained by double integration of the third Walsh function. We give an algorithm for constructing an interpolating spline of the second degree for a binary node system and find the approximation order of this interpolation process. We also prove that the system of dilations and shifts of the constructed B-spline generates an MRA (V_n) in De Boor sense. This MRA is not Riesz. But we can find the approximation order of functions from the Sobolev spaces $W_2^s, s > 0$ by the subspaces (V_n).

Key words: binary B-splines, multiresolution analysis, Sobolev spaces.

DOI: 10.18500/1816-9791-2018-18-2-172-182

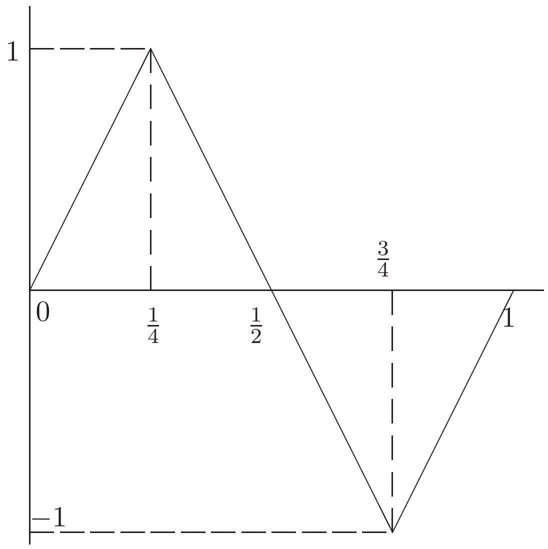

Fig. 1. The graph of the function $\frac{1}{4}\psi'(x)$

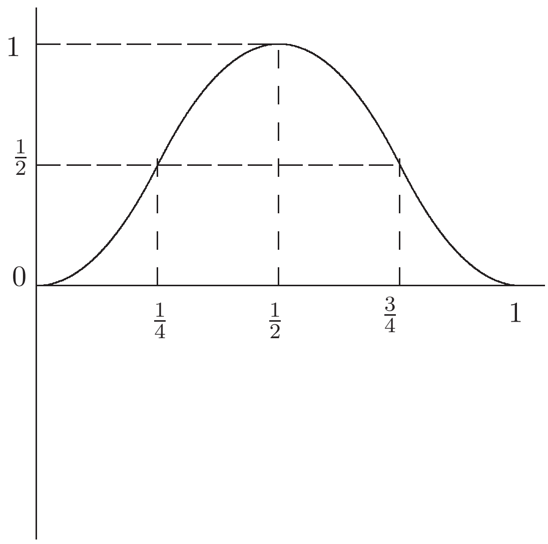

Fig. 2. The graph of the function $\psi(x)$

Acknowledgements: The authors are grateful to the referee for useful comments. This work was supported by the Russian Foundation for Basic Research (project no. 16-01-00152).

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Cite this article as:

Lukomskii S. F., Mushko M. D. On Binary B-splines of Second Order. *Izv. Saratov Univ. (N. S.), Ser. Math. Mech. Inform.*, 2018, vol. 18, iss. 2, pp. 172–182 (in Russian). DOI: 10.18500/1816-9791-2018-18-2-172-182.
