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On the Complex Dynamics in Simplest Vibrational Systems with Hereditary-Type Friction

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The dynamics of a number of vibrational systems, accounting for the forces of hereditary-type dry friction and a vibration limiter, are studied in the paper. The interaction between the vibration limiter and the vibrational system is assumed to obey Newton's hypothesis. A general mathematical model has been developed, which is a strongly nonlinear non-autonomous system with a variable structure. The dynamics of the mathematical model is studied numerically-analytically, using the mathematical apparatus of the point mapping method. The special feature of the studying approach is that a point map is not formed in a classical way (mapping Poincare surface into itself), but based on times of the relative rest of the vibrational system, which considerably simplified both the point mapping process and its detailed analysis. The presence of floating boundaries of plates of sliding motion required an original approach to point mapping and interpreting the results obtained. The developed investigation methodology and software product were used to study the phase-plane portrait of the mathematical model as a function of the characteristics of sliding friction forces and rest, as well as of the type and position of the limiter. Based on the character of the bifurcation diagrams variation, it was possible to find the main laws of the motion regimes alteration process (the occurrence of periodic motion regimes of arbitrary complexity and possible transition to chaos via the period-doubling process) with the changing parameters of the vibrational system (the amplitude and frequency of the periodic effect, forms of the functional relation describing the variation of the friction coefficient value of relative rest. The results obtained with and without accounting for a vibration limiter are also compared in the paper.

Key words: mathematical model, hereditary-type friction, Poincare function, relative rest, fixed point, chaos.

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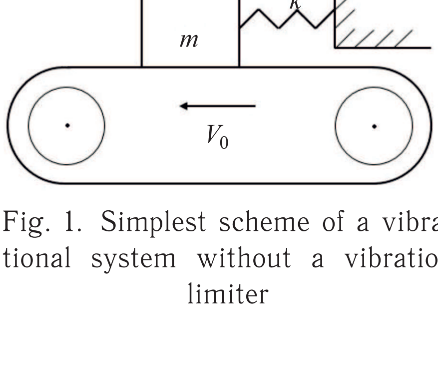


Fig. 1. Simplest scheme of a vibrational system without a vibration limiter

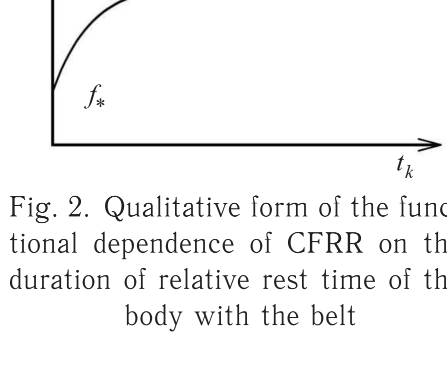


Fig. 2. Qualitative form of the functional dependence of CFRR on the duration of relative rest time of the body with the belt

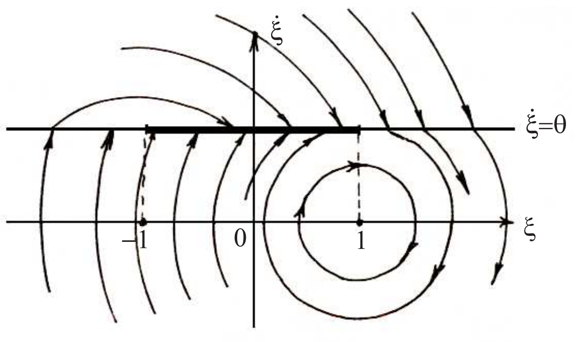


Fig. 3. Phase portrait of the simplest vibrational system

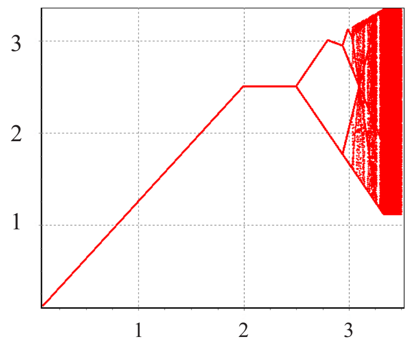


Fig. 4. Bifurcation diagram for the parameter ε^* for the simplest vibrational system

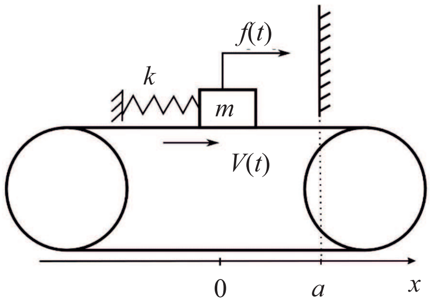


Fig. 5. Simplest scheme with a vibration limiter and external perturbation of the body with the belt

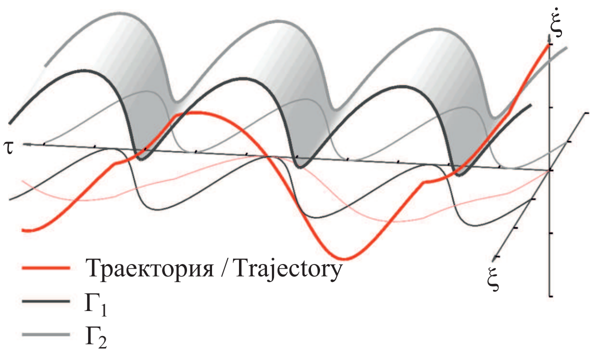


Fig. 6. Phase portrait of the simplest scheme of a vibrational system with a vibration limiter and external perturbation of the body and the belt

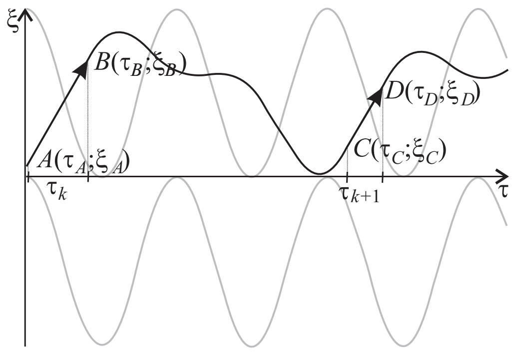
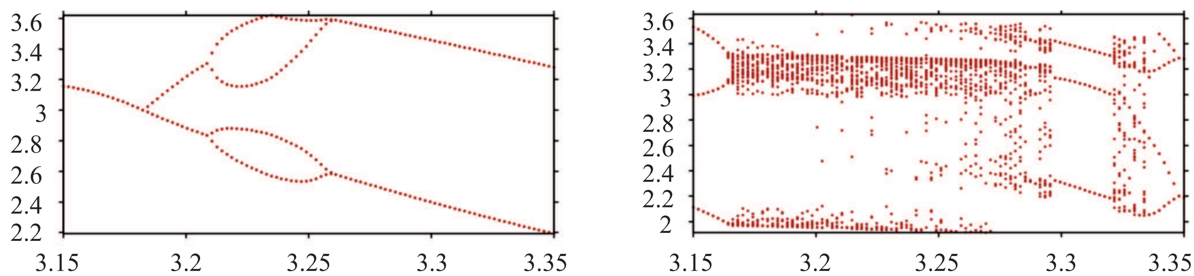


Fig. 7. Trajectories of the body motion during the relative rest time of the body and the belt as a function of time



a   *b*

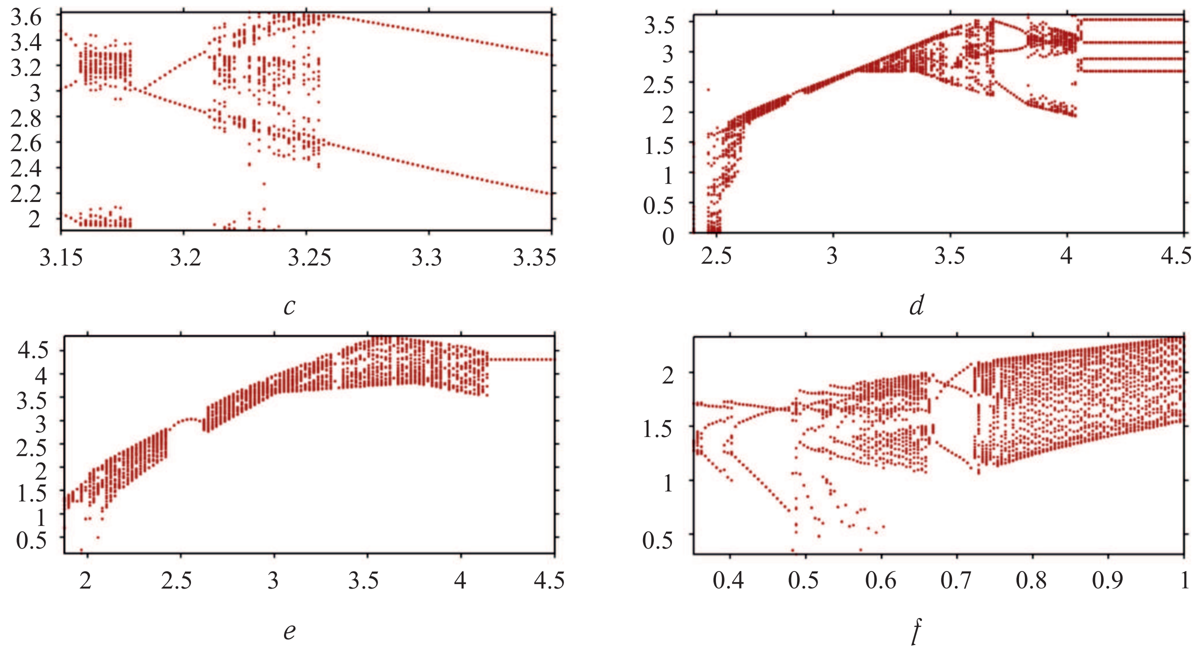


Fig. 8. Bifurcation diagrams for: *a* — constant external force and periodically changing velocity of the belt; *b*, *c* — constant external force and periodically changing velocity of the belt in the presence of a vibration limiter equal to 4.025 and 4.05, respectively; *d* — relative distance to the vibration limiter; *e* — coordinate of the vibration limiter for $R = 0.5$; *f* — recovery factor after the impact of the body against the vibration limiter

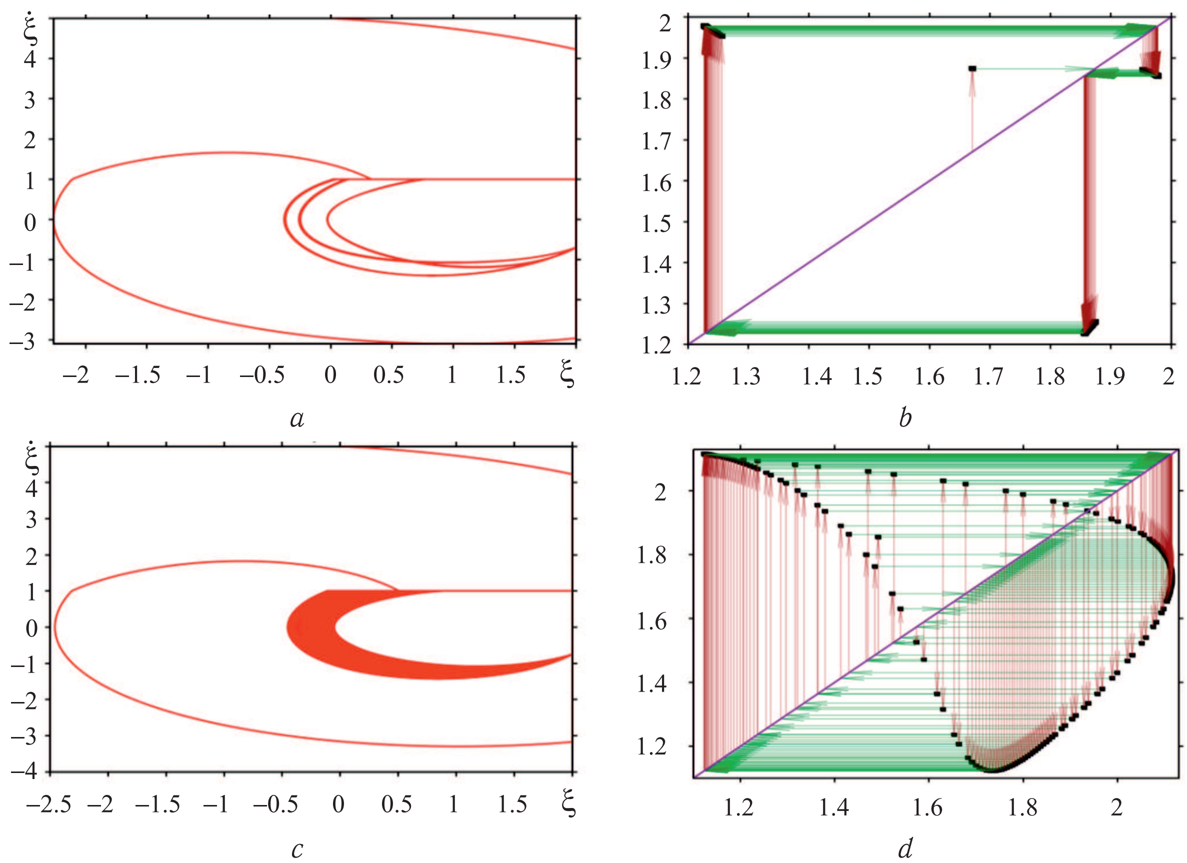
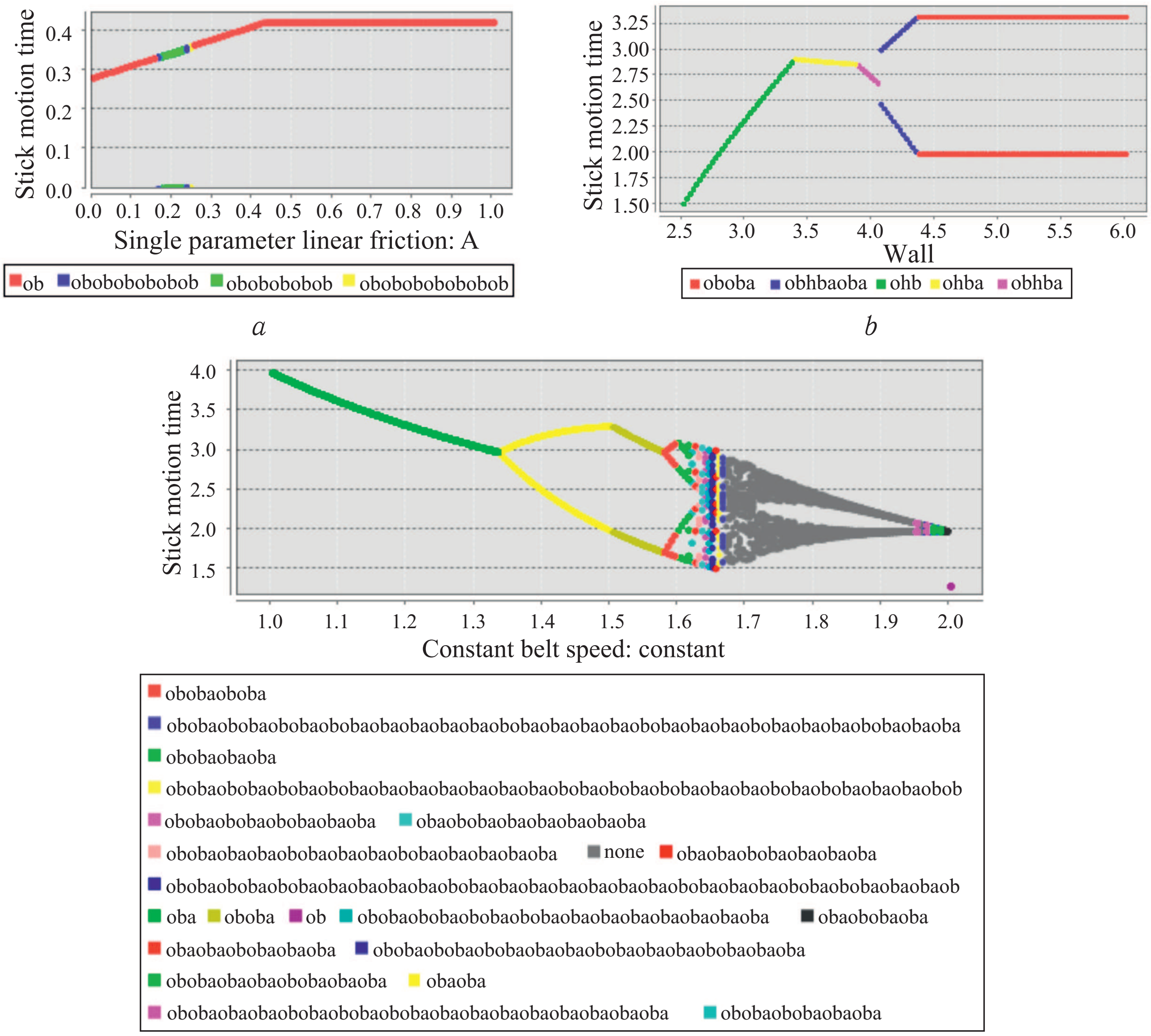


Fig. 9. Phase trajectories and Lameray diagrams for two values of the velocity recovery factor after the impact $R = 0.7$ (*a*, *b*) and $R = 0.75$ (*c*, *d*)



c

Fig. 10. Bifurcation diagrams: *a* — for the amplitude of external force; *b* — for the relative distance to the limiter; *c* — for the velocity of the belt

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