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One Counterexample of Shape-preserving Approximation

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Let $2s$ points $y_i = -\pi \le y_{2s} < \ldots < y_1 < \pi$ be given. Using these points, we define the points y_i for all integer indices i by the equality $y_i = y_{i+2s} + 2\pi$. We shall write $f \in \triangle^{(1)}(Y)$ if f is a 2π-periodic function and f does not decrease on $[y_i, y_{i-1}]$ if i is odd; and f does not increase on $[y_i, y_{i-1}]$ if i is even. We denote $E_n^{(1)}(f;Y)$ the value of the best uniform comonotone approximation. In this article the following counterexample of comonotone approximation is proved.

Example. For each $k \in \mathbb{N}$, $k > 2$, and $n \in \mathbb{N}$ there a function $f(x) := f(x; s, Y, n, k)$ exists, such that $f \in \triangle^{(1)}(Y)$ and

$$E_n^{(1)}(f;Y) > B_Y n^{\frac{k}{2}-1} \omega_k \left(f; \frac{1}{n}\right),$$

where B_Y =const, depending only on Y and k; ω_k is the modulus of smoothness of order k, of f.

Key words: trigonometric polynomials, polynomial approximation, shape-preserving.

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