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On Conditions for Distributivity or Modularity of Congruence Lattices of Commutative Unary Algebras

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The paper is devoted to the problem of describing unary algebras whose congruence lattices have a given property. By now this problem has been solved for algebras with one unary operation. In the paper it is shown that this problem is much more difficult for arbitrary commutative unary algebras. We give some necessary conditions for such lattices to be distributive or modular. Besides, it is proved here that a lattice of all subsets of a set is isomorphic to the congruence lattice of a suitable connected commutative unary algebra.

Key words: commutative unary algebra, distributive lattice, modular lattice, congruence lattice of an algebra.

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