UDC 517.95; 517.984

Classical Solution by the Fourier Method of Mixed Problems with Minimum Requirements on the Initial Data

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The article gives a new short proof the V. A. Chernyatin theorem about the classical solution of the Fourier method of the mixed problem for the wave equation with fixed ends with minimum requirements on the initial data. Next, a similar problem for the simplest functional differential equation of the first order with involution in the case of the fixed end is considered, and also obtained definitive results . These results are due to a significant use of ideas A. N. Krylova to accelerate the convergence of series, like Fourier series. The results for other similar mixed problems given without proof.

Key words: mixed problem, Fourier method, involution, classical solution, asymptotic form of eigenvalues and eigenfunctions, Dirac system.

This work was supported by the Russian Foundation for Basic Research (*project no.* 13-01-00238).

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