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About Generating Set of the Invariant Subalgebra of Free Restricted Lie Algebra

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Suppose that $L = L(X)$ is the free Lie p -algebra of finite rank k with free generating set $X = \{x_1, \dots, x_k\}$ on a field of positive characteristic. Let G is nontrivial finite group of homogeneous automorphisms $L(X)$. Our main purpose to prove that L^G subalgebra of invariants is infinitely generated. We have more strongly result. Let $Y = \cup_{n=1}^{\infty} Y_n$ be homogeneous free generating set for the algebra of invariants L^G , elements Y_n are of degree n relatively X , $n \geq 1$. Consider the corresponding generating function $\mathcal{H}(Y, t) = \sum_{n=1}^{\infty} |Y_n| t^n$. In our case of free Lie restricted algebras, we prove, that series $\mathcal{H}(Y, t)$ has a radius of convergence $1/k$ and describe its growth at $t \rightarrow 1/k - 0$. As a result we obtain that the sequence $|Y_n|$, $n \geq 1$, has exponential growth.

Key words: free Lie algebras, Lie p -algebras, invariants, generating set.

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