



UDC 511.325

Distribution of Values of Dirichlet Characters in the Sequence of Shifted Primes

Z. Kh. Rakhmonov

Institute of Mathematics, Academy of Sciences of the Republic of Tajikistan, 734063, Dushanbe, Ayni st., 83, rakhmonov-r@rambler.ru

The new estimate for the sum of the values of a primitive Dirichlet character modulo an integer q has been obtained over the sequence of shifted primes $p - l$, $(l, q) = 1$, $p \leq x$. This estimate is nontrivial for $x \geq q^{\frac{5}{6} + \varepsilon}$ and refines the estimate obtained by J. B. Friedlander, K. Gong, I. E. Shparlinskii. Their estimate holds provided that $x \geq q^{8/9 + \varepsilon}$.

Key words: Dirichlet character, shifted primes, short sums of characters, exponential sums over primes.

References

1. Vinogradov I. M. On the distribution of quadratic rests and non-rests of the form $p + k$ to a prime modulus. *Rec. Math. Moscow, n. Ser.*, 1938, vol. 3, no. 45, pp. 311–319 (in Russian).
2. Vinogradov I. M. An improvement of the estimation of sums with primes. *Bull. Acad. Sci. URSS. Ser. Math.* [Izvestia Akad. Nauk SSSR] 1943, vol. 7, pp. 17–34 (in Russian).
3. Vinogradov I. M. New approach to the estimation of a sum of values of $\chi(p + k)$. *Izvestiya Akad. Nauk SSSR. Ser. Mat.*, 1952, vol. 16, pp. 197–210 (in Russian).
4. Vinogradov I. M. Improvement of an estimate for the sum of the values $\chi(p + k)$. *Izvestiya Akad. Nauk SSSR. Ser. Mat.*, 1953, vol 17, pp. 285–290 (in Russian).
5. Vinogradov I. M. An estimate for a certain sum extended over the primes of an arithmetic progression. (*Russian*) *Izv. Akad. Nauk SSSR Ser. Mat.*, 1966, vol 30, no. 3, pp. 481–496 (in Russian).
6. Karatsuba A. A. Sums of characters, and primitive roots, in finite fields. *Doklady Akademii Nauk SSSR*, 1968, vol 180, no. 6, pp. 1287–1289 (in Russian).
7. Karatsuba A. A. Estimates of character sums. *Math. USSR-Izv.*, 1970, vol. 4, no. 1, pp. 19–29.
8. Karatsuba A. A. Sums of characters over prime numbers. *Math. USSR-Izv.*, 1970, vol. 4, no. 2, pp. 303–326.
9. Rakhmonov Z. Kh. On the distribution of values of Dirichlet characters. *Rus. Math. Surv.*, 1986, vol. 41, no. 1, pp. 237–238. DOI: 10.1070/RM1986v041n01ABEH003232.
10. Rakhmonov Z. Kh. Estimation of the sum of characters with primes. *Dokl. Akad. Nauk Tadzhik. SSR*, 1986, vol 29, no. 1, pp. 16–20 (in Russian).
11. Rakhmonov Z. Kh. On the distribution of the values of Dirichlet characters and their applications. *Proc. Steklov Inst. Math.*, 1995, vol. 207, no. 6, pp. 263–272.
12. Fridlander Dzh. B., Gong K., Shparlinskii I. E. Character sums over shifted primes. *Math. Notes*, 2010, vol. 88, iss. 3–4, pp. 585–598. DOI: 10.1134/S0001434610090312.
13. Rakhmonov Z. Kh. A theorem on the mean value of $\psi(x, \chi)$ and its applications. *Russian Academy of Sciences. Izvestiya Mathematics*, 1994, vol. 43, no. 1, pp. 49–64. DOI: 10.1070/IM1994v043n01ABEH001558.
14. Rakhmonov Z. Kh. A theorem on the mean-value of Chebyshev functions. *Russian Academy of Sciences. Izvestiya Mathematics*, 1995, vol. 44, no. 3, pp. 555–569. DOI 10.1070/IM1995v044n03ABEH001613.
15. Vinogradov A. I. On numbers with small prime divisors. *Dokl. Akad. Nauk SSSR*, 1956, vol 109, no. 4, pp. 683–686 (in Russian).
16. Burgess D. A. On character sum estimate with $r = 3$. *J. London Math. Soc.*, 1986, vol. 33, no. 2, pp. 219–226. DOI: 10.1112/jlms/s2-33.2.219.