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Article

Modeling the reliability of the onboard equipment of a mobile robot

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Abstract. Mobile robots with complex onboard equipment are investigated in this article. It is shown that their onboard equipment, for providing the required reliability parameters, must have fault-tolerant properties. For designing such equipment it is necessary to have an adequate model of reliability parameters evaluation. The approach, linked to the creation of the model, based on parallel semi-Markov process apparatus, is considered. At the first stage of modeling, the lifetime of the single block in a complex fault-recovery cycle is determined. Dependences for the calculation of time intervals and probabilities of wandering through ordinary semi-Markov processes for a common case are obtained. At the second stage, ordinary processes are included in the parallel one, which simulates the lifetime of the equipment lifetime as a whole. To simplify calculations, a digital model of faults with the use of the procedure of histogram sampling is proposed. It is shown that the number of samples permits to control both the accuracy and the computational complexity of the procedure for calculating the reliability parameters.

Keywords: reliability, failure, fault-tolerance, semi-Markov process, sampling, modeling, accuracy, computational complexity

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Моделирование надежности бортового оборудования мобильного робота

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Аннотация. Исследованы мобильные роботы со сложным бортовым оборудованием. Показано, что бортовое оборудование для обеспечения требуемых параметров надежности должно обладать отказоустойчивыми свойствами, а для проектирования такого оборудования необходима адекватная модель оценивания его надежности. Рассмотрен подход, связанный с созданием модели, основанной на теории параллельных полумарковских процессов. На первом этапе моделирования определяется срок службы единственного блока в сложном цикле устранения неисправностей. Получены зависимости для расчета временных интервалов и вероятностей блуждания по обычным полумарковским процессам для общего случая. На втором этапе обычные процессы включаются в параллельный, который имитирует срок службы оборудования в целом. Для упрощения расчетов предложена цифровая модель неисправностей с использованием процедуры построения гистограмм. Показано, что количество выборок позволяет контролировать как точность, так и вычислительную сложность процедуры расчета параметров надежности.
Ключевые слова: надежность, отказ, отказоустойчивость, полумарковский процесс, выборка, моделирование, точность, вычислительная сложность
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Introduction

Mobile robots that execute target tasks in an aggressive environment are currently widely used in industry, anti-terrorism operations, technology disaster consequences elimination, military sphere, etc. [1–3]. The impact of the environment leads to the fact that the robot's onboard equipment reliability indicator, namely mean time between failures, falls sharply, which reduces the robot's lifetime in general. To increase the lifetime, the planned redundancy is introduced into the equipment. Since the reliability parameters [4–6] of an individual block are limited, this problem can be solved only systematically, using redundant fault-tolerant structures [7–9]. For proper planning the redundancy it is necessary to simulate the failure-recovery process of both only units and equipment as a whole preliminary. The general approach to modeling the reliability of a system is based on the theory of Markov [10,11] or semi-Markov [12–15] processes which allow describing a single unit of equipment lifetime. Using more rough Markov models instead of semi-Markov ones, we downgrade the accuracy of the simulation procedure. Other approaches to simulation suppose the application of the Monte-Carlo method [16], the chaos expansion method [17], the graph theory [18, 19] but all approaches are insufficient, due to the fact that they do not take into account that in redundant structures a competition effect arises. Below it is proposed to use discrete semi-Markov models instead of Markov models to describe the competition in fault-tolerant assemblies in which accuracy can be controlled by changing the number of samples at distribution densities. Therefore, it is necessary to develop a model whose accuracy can be estimated and increased/decreased in accordance with the solvable reliability problem, which explains the necessity and relevance of this study.

1. The approach to simulation of fault-tolerant systems

Mobile robot equipment, in which the fault-tolerance principle is realized, may be considered as M units, operated in parallel [20]. Fault/recovery processes in assembly units develop in parallel, so such an abstraction as M-parallel semi-Markov process [21] may be obtained to describe the reliability of the assembly as follows:

$$\boldsymbol{\mu} = [\mu_1, ..., \mu_m, ..., \mu_M] .$$

where μ_m, $1 \leqslant m \leqslant M$ is the ordinary semi-Markov process [12–14], which is characterized with a set of states $A_m = \left\{a_{0(m)}, ..., a_{j(m)}, ..., a_{J(m)}\right\}$ and a semi-Markov matrix $\boldsymbol{h}_m(t) = \left[h_{j(m),k(m)}(t)\right]$;

$$\mu_m = \{A_m, \boldsymbol{h}_m(t)\},$$

where t is the time $a_{0(m)}$; simulates the start of m-th unit exploiting, when it is surely able to work; $a_{J(m)}$ is the absorbing state, which simulates the fully destroyed unit; $a_{j(m)}$, $1(m) \leqslant j(m) < J(m)$ simulate other physical states (able to work, short-time failures, under recovering, etc.);

$$\boldsymbol{h}_m(t) = \boldsymbol{p} \otimes \boldsymbol{f}_m(t); \tag{1}$$

where $\boldsymbol{p}_m = \left[p_{j(m),k(m)}\right]$ and $\boldsymbol{f}_m(t) = \left[f_{j(m),k(m)}(t)\right]$ are $[J_m + 1] \times [J_m + 1]$ stochastic matrix and matrix of pure time densities, correspondingly.

Semi-Markov matrix (1) has the following features: elements of the matrix $\boldsymbol{h}_m(t)$ zero column, J_m-th row and diagonal elements are equal to zeros. Physically it means, that any unit cannot return to the beginning of exploiting, cannot return from the state of complete destruction and cannot switch to the same state, as before switching. Weighted time densities $h_{j(m),k(m)}(t)$ describe both sojourn time in the state $a_{j(m)}$, and prior probabilities of switching into conjugative states. Due to there is the only absorbing state in μ_m, for elements of rows from 0_m-th till $[J_{m-1}]$-th the next expression is true:

$$\sum_{k(m)=1(m)}^{J(m)} \int_0^\infty h_{j(m),k(m)}(t)\,dt = 1, \quad 0(m) \leqslant j(m) \leqslant J(m).$$

Both probabilities $p_{j(m),k(m)}$ and parameters of time densities $f_{j(m),k(m)}(t)$, such as expectations and dispersions, defining fault/recovery process in m-th unit, depend on the material, of which the element is made, quality of element manufacturing and assembling, exploiting conditions, side effects, and so on, and define mobile robots' reliability in common.

From the common problem of reliability estimation digitizing may be set off three tasks, which one should fulfill:

– the estimation of the time till failure (random walk from $a_{0(m)}$ till $a_{J(m)}$);

– the estimation of the time and transition probability of random walk from arbitrary $a_{j(m)} \neq a_{0(m)} \neq a_{j(M)}$ till arbitrary $a_{k(m)} \neq a_{0(m)} \neq a_{j(M)}$;

– the estimation of the time and probability of returning to $a_{j(m)} \neq a_{0(m)} \neq a_{j(M)}$.

In general, time till failure may be defined as follows [22]:

$$\tilde{f}_{0(m),J(m)}(t) = L^{-1}\left[I^R_{0(m)} \cdot \sum_{w=1}^{\infty} \{L(h_m(t))\}^w \cdot I^C_{J(m)}\right], \tag{2}$$

where $I^R_{0(m)}$ is the $[J(m)+1]$-size row-vector, in which $0(m)$-th element is equal to one, and other elements are equal to zeros; $I^C_{J(m)}$ is the $[J(m)+1]$-size column-vector, in which $J(m)$-th element is equal to one, and other elements are equal to zeros; L and L^{-1} are direct and inverse Laplace transforms, correspondingly. To solve the second task one should transform $\boldsymbol{h}_m(t)$ as follows

$$\boldsymbol{h}_m(t) \to \boldsymbol{h}'_m(t).$$

During the transformation, the only restriction imposed onto wandering trajectories is that neither $a_{j(m)}$, nor $a_{k(m)}$ state processes should fall twice. To form $\boldsymbol{h}_m(t)'$ with such properties in semi-Markov matrix $\boldsymbol{h}_m(t)$ all elements of $j(m)$-th column and $k(m)$-th row should be replaced by zeros. Elements $h_{i(m),l(m)}(t)$ should be recalculated as follows:

$$h'_{i(m),l(m)}(t) = \frac{h_{i(m),l(m)}(t)}{\sum\limits_{\substack{k(m)=0(m),\\ k(m)\neq j(m)}}^{J(m)} p_i(m),k(m)}, \quad 0(m) \leqslant i(m), j(m), k(m) \leqslant J(m), \quad i(m) \neq k(m).$$

Stochastic summation of densities, formed on all possible wandering trajectories, gives the following expression:

$$\tilde{h}'_{j(m),k(m)}(t) = I^R_{j(m)} \cdot L^{-1}\left[\sum_{w=1}^{\infty} \{L[h_m(t)']\}^w\right] \cdot I^C_{k(m)}, \tag{3}$$

where $I^R_{j(m)}$ is the row-vector, in which $j(m)$-th element is equal to one, and other elements are equal to zeros; $I^C_{k(m)}$ is the column-vector, in which $k(m)$-th element is equal to one, and other elements are equal to zeros.

In the semi-Markov process $\boldsymbol{h}_m(t)'$ there are, as a minimum, two absorbing states, namely $a_{k(m)}$ and $a_{j(m)}$, so the group of events of reaching from is not full, and in common case $\tilde{h}'_{j(m),k(m)}(t)$ is weighted, but not pure density. The state $a_{k(m)}$ from the state $a_{j(m)}$ may be reached with probability [23]

$$\tilde{p}'_{j(m),k(m)} = \int_0^\infty \tilde{h}'_{j(m),k(m)}(t)\,dt$$

and pure time density of wandering from the state $a_{j(m)}$ to the state $a_{k(m)}$ may be defined as follows:

$$\tilde{f}'_{j(m),k(m)}(t) = \frac{\tilde{h}'_{j(m),k(m)}(t)}{\tilde{p}'_{j(m),k(m)}(t)}, \tag{4}$$

when solving the third task, one should execute the following transformation:

$$\boldsymbol{h}_m(t) \to \boldsymbol{h}''_m(t),$$

where one row and one column are added to the matrix; complementary, $[J(m)+1]$-th row should be fulfilled with zeros; $j(m)$-th column at first should be carried over the complementary -th column, and then it should be fulfilled with zeros. Stochastic summation of densities, formed on all possible wandering trajectories, gives next expression:

$$\tilde{h}''_{j(m),k(m)}(t) = I^R_{j(m)} \cdot L^{-1}\left[\sum_{w=1}^{\infty}\{L[h_m(t)'']\}^w\right] \cdot I^C_{J(m)+1}, \tag{5}$$

where $I^R_{j(m)}$ is the $[J(m)+2]$-size row-vector, in which j_m-th element is equal to one, and other elements are equal to zeros; $I^C_{J(m)+1}$ is the $[J(m)+2]$-size column-vector, in which $[J(m)+1]$-th element is equal to one, and other elements are equal to zeros. In the semi-Markov process $\boldsymbol{h}''_m(t)$ there are two absorbing states, namely $a_{J(m)}$ and $a_{J(m)+1}$, so the group of events of reaching $a_{J(m)+1}$ from $a_{j(m)}$ is not full and in common case $\tilde{h}''_{j(m),J(m)+1}(t)$ is weighted, but not pure density. The state $a_{J(m)+1}$ from the state $a_{j(m)}$ may be reached with probability

$$\tilde{p}''_{j(m),J(m)+1} = \int_0^\infty \tilde{h}''_{j(m),J(m)+1}(t)\,dt$$

and during pure time density

$$\tilde{f}''_{j(m),J(m)+1}(t) = \frac{\tilde{h}''_{j(m),J(m)+1}(t)}{\tilde{p}''_{j(m),J(m)+1}(t)}. \tag{6}$$

2. Sampling of time densities

As it follows from (2), (3), (5), expressions for calculation of densities $\tilde{f}_{0(m),J(m)}(t)$, $\tilde{f}'_{j(m),k(m)}(t)$, $\tilde{f}''_{j(m),J(m)+1}(t)$ are too complicated to use them for analysis of robotic system reliability, due to the fact there is so-called "competition" [20, 21] for failure among equipment units. So, to investigate the reliability of the robotic system as a whole, one should use any approach to densities mentioned.

Let us consider generalized density

$$\phi_m(t) \in \{\tilde{f}_{0(m),J(m)}(t), \tilde{f}'_{j(m),k(m)}(t), \tilde{f}''_{j(m),J(m)+1}(t)\}.$$

In the most general case $\phi_m(t)$ is a continual function with the next common properties:

$$0 \leqslant t_{\min} \leqslant \arg[\phi_m(t)] \leqslant t_{\max} < \infty.$$

Time density $\phi_m(t)$ may be represented as a histogram. For this purpose domain $[t_{\min}, t_{\max}]$ should be divided into X intervals, $0 \leqslant t < \tau_1$, ...,$\tau_{x-1} \leqslant t < \tau_x$, ...,$\tau_{X-1} \leqslant t < \infty$, as it is shown in the Figure. For simplification of the model, it is advisable to do both borders τ_x between histogram intervals and sampling points θ_x representing intervals, uniform for all $1 \leqslant m \leqslant M$.

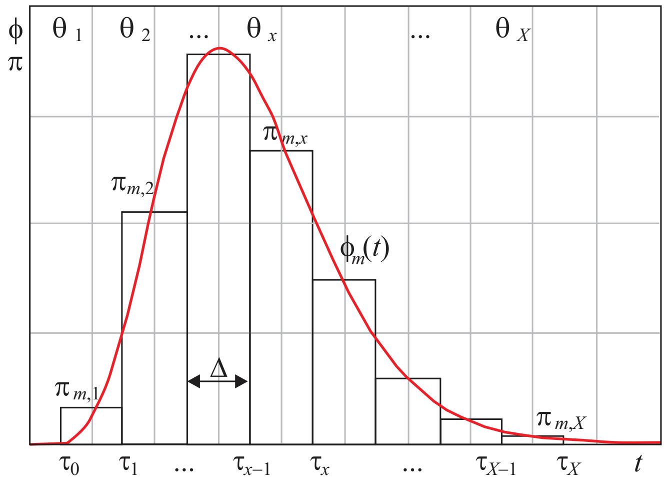

Figure. Time density sampling

Histogram intervals width D is as follows:

$$\Delta = \frac{\tau_1 - \tau_{X-1}}{X-2}, \tag{7}$$

where X is the quantity of histogram intervals; τ_1 and τ_{X-1} are the right border of the first interval and the left border of the X-th interval.

Values of intervals are equal to

$$\pi_{m,x} = \int_{l(x)}^{r(x)} \phi_m(t)\, dt, \tag{8}$$

where $l(x) \leqslant t \leqslant r(z)$ are left and right limits of integration interval;

$$l(x) = \{\tau_1 + \Delta(x-2)\}, \quad \text{when } 2 \leqslant x \leqslant X,$$
$$r(x) = \{\tau_1 + \Delta(x-1)\}, \quad \text{when } 1 \leqslant x \leqslant X-1.$$

Sampling points, representing histogram intervals, are as follows

$$\theta_x = \tau_x - \frac{\Delta}{2}.$$

In a discrete model, every interval of the histogram is represented as weighted shifted degenerative distribution law, so the time density of the described histogram is as follows:

$$\tilde{\phi}_m(t) = \sum_{x=1}^{X} \pi_{m,x} \cdot \delta(t - \theta_x), \tag{9}$$

where $\delta(...)$ is the shifted Dirac δ-function;$\pi_{m,x}$ is the weight of Dirac function:

$$\sum_{x=1}^{X} \pi_{m,x} = 1.$$

The error of digitizing may be estimated as follows:

$$\epsilon_m = \int_0^{l(0)} \phi_m(t)\,dt + \sum_{x=1}^{X} \int_{l(x)}^{r(x)} |\phi_m(t) - \pi_{m,k}|\,dt + \int_{\tau(K)}^{\infty} \phi_m(t)\,dt.$$

One would admit that there are no restrictions, imposed on $\phi_m(t)$, besides $\arg[\phi_m(t)] \geqslant 0$. Expression (9) satisfies this restriction so process μ still remains the semi-Markov one.

3. Interaction in fault-tolerant system

Mobile robot redundant units, assembled into the fault-tolerant structures during operation, compete for failure. The result of this competition is the failure of m-th unit the first or not the first. The lifetime of M-units redundant structure is as follows [20, 21]:

$$\phi_\nu(t) = \frac{d\{1 - \prod_{m=1}^{M} [1 - \Phi_m(t)]\}}{dt}, \tag{10}$$

where $\Phi_m(t)$ is the distribution function;

$$\Phi_m(t) = \int_0^t \phi_m(\xi)\,d\xi. \tag{11}$$

The weighted time density, the probability, and the pure time density of winning the competition for failure by the m-th unit are as follows

$$\vartheta_{\nu,m}(t)\phi_m(t) \prod_{\substack{l=1 \\ l\neq m}}^{M} [1 - \Phi_l(t)], \quad \pi_\nu(m) = \int_0^\infty \vartheta_{\nu,m}(t)\,dt, \quad \phi_{\nu,m}(t) = \frac{\vartheta_{\nu,m}(t)}{\pi_{\nu,m}}, \tag{12}$$

where $\vartheta_{\nu,m}(t)$ is the weighted time density; $\pi_{\nu,m}$ is the probability; $\phi_{\nu,m}(t)$ is the pure time density of winning by the m-th unit.

When $\phi_m(t)$ is transformed into its discrete analog $\tilde{\phi}_m(t)$, as it is shown at (9), the time distribution function is transformed to $\tilde{\Phi}_m(t)$:

$$\tilde{\Phi}_m(t) = \int_0^t \tilde{\phi}_m(\xi)\,d\xi = \sum_{x=1}^{X} \pi_{m,x} \cdot \eta(t - \theta_x), \tag{13}$$

where $\eta(t - \theta_x)$ is the shifted Heaviside function.

Dependence (12) may be transformed into the sequence of samples as follows:

$$\tilde{\Phi}_m(t) \to \overline{\Phi}_m(t) = \sum_{x=1}^{X} [(\sum_{y=1}^{x} \pi_{m,y}) \cdot \delta(t - \theta_x)], \tag{14}$$

where $\sum_{y=1}^{x} \pi_{m,y}$ is the nomination of the function $\overline{\Phi}_m(t)$ sample at the point θ_x.

Accordingly, $1 - \Phi_m(t)$ function may be transformed into discrete form as follows:

$$\overline{[1 - \Phi_m(t)]} = \sum_{x=1}^{X} [(\sum_{y=x+1}^{X} \pi_{m,y}) \cdot \delta(t - \theta_x)].$$

Combinations of $\tilde{\phi}_m(t)$ and $\overline{[1 - \Phi_m(t)]}$ permit us to construct the discrete analog $\overline{\vartheta}_{\nu,m}(t)$ of function (10). It is necessary to admit, that when time intervals are described with continual functions $\phi_m(t)$,$1 \leqslant m \leqslant M$, then there may be only one winner in the competition(10) due to the fact, that probability of competition draw, even in the case of paired races, is too small in comparison with probabilities of winning by one of the participants. When time intervals are described with discrete distribution function, a draw effect emerges with probabilities, comparable with winnings and losing probabilities due to the fact, that time interval $\tau_{x-1} \leqslant t < \tau_x$ may include number of events. To determine possible combinations $\tilde{\phi}_m(t)$ and $\overline{[1 - \Phi_m(t)]}$ it is necessary to consider data, which includes M binary digits:

$$n = \langle n(1), ..., n(m), ..., n(M) \rangle,$$

where inside triangle brackets there is the code, obtained by means of Cartesian exponentiation to M-th degree the set $[0, 1]$; $n(m) \in [0, 1]$ is binary digit $0 \leqslant n < 2^n$.

All codes n may be gathered onto set N, which is divided onto subsets N_l:

$$N = N_0, ..., N_l, ..., N_M,$$

where N_l is the subset of codes, which include l “nulls” and M-l “ones”. In turn,

$$N_l = \{n_{1(M,l)}, ..., n_{c(M,l)}, ..., n_{C(M,l)}\},$$

where $n_{c(M,l)}$ is $c(M, l)$-th M-digits code, including l “nulls” and $M - l$ “ones”; $C(M, l)$ is common quantity of such codes; $c(M, l)$ is the index, which numerates codes in the set N_l;

$$C(M, l) = \frac{M!}{l! \cdot (M - l)!},$$
$$n_{c(M,l)} = \langle n[1, c(m, L)], ..., n[m, c(m, L)], ..., n[M, c(m, L)] \rangle,$$
$$n[M, c(m, L)] \in [0, 1]. \quad (15)$$

The function of two parameters, namely, time and m-th code digit state $n[m, c(M, l)]$ should be introduced to describe distribution:

$$\psi\{t, n[m, c(M, l)]\} = \{\tilde{\phi}_m(t) \quad \text{when } n[m, c(M, l)] = 0\}. \quad (16)$$

A competition outcome, alike (11), when l units of M failure during the time interval $\tau_{x-1} \leqslant t < \tau_x$, may be expressed as

$$\tilde{\vartheta}_{\nu,l/M}(t) = \sum_{m=1}^{M} \prod_{c(M,l)=1}^{C(M,l)} \psi\{t, n[m, c(M, l)]\}.$$

The probability and pure discrete time distribution and mean time of l/M units simultaneous failure are as follows:

$$\tilde{\pi}_{\nu,l/M} = \int_0^\infty \tilde{\vartheta}_{\nu,l/M}(t)dt, \quad \tilde{\phi}_{\nu,l/M}(t) = \frac{\tilde{\vartheta}_{\nu,l/M}(t)}{\tilde{\pi}_{\nu,l/M}}, \quad \tilde{\pi}_{\nu,l/M} = \int_0^\infty t \cdot \tilde{\tau}_{\nu,l/M}(t)\, dt.$$

Let some robot fault-tolerant assemble to be workable until among all M units at least one unit stays "alive". There are 2^{M-1} combinations of reaching unworkable state, e. g., 3-units assemble may fail as $1+1+1$, $1+2$, $2+1$, 3. Analysis of every combination gives different probabilities and pure time densities for evaluation of assemble "lifetime". So it is necessary to evaluate time till failure for every combination, and then stochastically summarize them.

4. Digital calculation of reliability parameters

The above theoretical calculations follow the digital method of fault-tolerant system reliability parameters estimation.

1. Working out the model of single unit failure/recovery process and calculation time density till failure of this unit accordingly (2), (4), (6).

2. Transformation of time density into discrete form accordingly (7), (8).

3. With use of the formulae (13), (14), (15), (16), calculation discrete distribution of assembling "lifetime" for different combinations of units failures/recoveries.

4. Estimation of reliability parameters of the fault-tolerant assembles as a whole.

Conclusion

As a result, the task of designing fault-tolerant assemblies was proposed to be divided into two stages:

– development of conventional semi-Markov models of individual units, and conversion it to the discrete form;

– analysis of a parallel discrete semi-Markov process to obtain the reliability parameters of equipment as a whole.

The proposed approach allows us to create a model of a redundant system with any degree of accuracy, to develop a method for optimizing a fault-tolerant system based on the approach of a discrete model. Further research in this area may be aimed at modeling many practical redundant systems with complex interactions between components and complex "life cycle" algorithms.

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