Известия Саратовского университета. Новая серия. Серия: Математика. Механика. Информатика. 2023. Т. 23, вып. 2. С. 264–273
Izvestiya of Saratov University. Mathematics. Mechanics. Informatics, 2023, vol. 23, iss. 2, pp. 264–273
mmi.sgu.ru https://doi.org/10.18500/1816-9791-2023-23-2-264-273, EDN: BTQTSM

Article

Optimal solution in the model of control over an economic system in the condition of a mass disease

I. V. Lutoshkin✉, **M. S. Rybina**

Ulyanovsk State University, 42 Leo Tolstoy St., Ulyanovsk 432017, Russia

Igor V. Lutoshkin, lutoshkiniv@ulsu.ru, https://orcid.org/0000-0002-4108-7646, AuthorID: 12324
Maria S. Rybina, rybina_maria@icloud.com, AuthorID: 1185864

Abstract. In the conditions of a mass disease, governing bodies of an economic system face a number of tasks related to the need to minimize its negative effects. This requires a tool that allows timely predicting the dynamics of the situation and determining what measures need to be taken. In this paper, a specialized mathematical model is proposed as such a tool, taking into account socio-biological and economic factors. The model is a dynamic optimal control problem with a delay in phase variables. The values of the model parameters were estimated using statistical data on the COVID-19 pandemic in the Russian Federation and the Ulyanovsk region. As target functionals, the following are considered: "social criterion" — a decrease in the number of cases; "economic criterion" — an increase in the relative profit of an economic system. To solve the problem, the authors apply a modification of the numerical parameterization method developed in their early studies. The article presents and analyzes the results of the numerical experiment aimed at studying the obtained optimal solutions. It is shown that: the optimal solution for social and economic criteria when changing budgets is stable; most of the parameters of the optimal solution are weakly elastic relative to the values of variables considered; the parameters of the optimal solution when using the economic criterion are more susceptible to change than when using the social criterion; the nature of the change in the parameters of the optimal solution for the Ulyanovsk region and for the Russian Federation is similar. Thus, the paper offers a tool for analyzing an economic problem in conditions of mass disease and confirms the applicability of the tool for finding optimal management strategies in various economic systems.

Keywords: optimal control, mathematical modeling, optimal solution analysis, economic system, mass disease, COVID-19

For citation: Lutoshkin I. V., Rybina M. S. Optimal solution in the model of control over an economic system in the condition of a mass disease. *Izvestiya of Saratov University. Mathematics. Mechanics. Informatics*, 2023, vol. 23, iss. 2, pp. 264–273. https://doi.org/10.18500/1816-9791-2023-23-2-264-273, EDN: BTQTSM

Научная статья
УДК 517.977.5,519.863

Оптимальное решение в модели управления экономической системой в условиях массового заболевания

И. В. Лутошкин✉, **М. С. Рыбина**

Ульяновский государственный университет, Россия, 432017, г. Ульяновск, ул. Льва Толстого, д. 42

Лутошкин Игорь Викторович, кандидат физико-математических наук, заведующий кафедрой цифровой экономики, lutoshkiniv@ulsu.ru, https://orcid.org/0000-0002-4108-7646, AuthorID: 12324
Рыбина Мария Сергеевна, магистрант факультета математики, информационных и авиационных технологий, rybina_maria@icloud.com, AuthorID: 1185864

Аннотация. В условиях массового заболевания перед органами управления экономической системы возникает ряд задач, связанных с необходимостью минимизировать его негативные эффекты. Для этого требуется инструмент, позволяющий оперативно прогнозировать динамику ситуации и определять, какие меры требуется принять. В данной работе в качестве такого инструмента предлагается специализированная математическая модель, учитывающая социально-биологические и экономические факторы. Модель представляет собой динамическую задачу оптимального управления с запаздыванием по фазовым переменным. Значения параметров модели оценены с использованием статистических данных о пандемии COVID-19 в Российской Федерации и Ульяновской области. В качестве целевого функционала рассматриваются: «социальный критерий» — уменьшение количества заболевших; «экономический критерий» — увеличение относительной прибыли экономической системы. Для решения задачи авторами применяется модификация численного метода параметризации, развиваемого ими в ранних исследованиях. В статье представлены и проанализированы результаты численного эксперимента, направленного на исследование полученных оптимальных решений. Показано, что оптимальное решение для социального и экономического критериев при изменении бюджетов является устойчивым; большинство параметров оптимального решения слабо эластичны относительно рассмотренных значений переменных; параметры оптимального решения при использовании экономического критерия более подвержены изменению, чем при использовании социального критерия; характер изменения параметров оптимального решения для Ульяновской области и для РФ является схожим. Таким образом, в работе предлагается инструмент анализа экономической проблемы в условиях массового заболевания, подтверждается применимость инструмента для поиска оптимальных стратегий управления в различных экономических системах.

Ключевые слова: оптимальное управление, математическое моделирование, анализ оптимального решения, экономическая система, массовое заболевание, COVID-19

Для цитирования: *Lutoshkin I. V., Rybina M. S.* Optimal solution in the model of control over an economic system in the condition of a mass disease [*Лутошкин И. В., Рыбина М. С.* Оптимальное решение в модели управления экономической системой в условиях массового заболевания] // Известия Саратовского университета. Новая серия. Серия: Математика. Механика. Информатика. 2023. Т. 23, вып. 2. С. 264–273. https://doi.org/10.18500/1816-9791-2023-23-2-264-273, EDN: BTQTSM

Introduction

The globalization of economy and growth of the world population lead to the fact that mass diseases have an impact on all aspects of society. In particular, they affect the social sphere, since morbidity increases negative processes in society and in some cases can lead to a reduction in the population [1]. In addition, changes are taking place in the economic sphere: with the introduction of restrictive measures to prevent the growth of morbidity, the business activity of economic entities decreases, labor productivity decreases [2]. Under the current conditions, a governing body of an economic system faces a number of tasks related to the need to minimize negative effects of a mass disease. Such tasks include: determining the amount of expenses for hospitalization of patients and providing them with medical care, informing citizens about the disease and ways to combat it, making a decision on the introduction of restrictive measures [3]. For their effective solution, a tool is needed to quantify the impact of the disease on socio-economic factors and determine a management strategy in conditions mentioned above. An example of a mass disease affecting the economy of the whole world is the COVID-19 pandemic. Due to the speed of the spread of the disease, it became necessary to promptly make decisions on the allocation of resources for health and social support, and on the need to introduce restrictive measures. As an effective tool for implementing these actions, a mathematical model describing this situation can act. In current publications devoted to modeling the dynamics of mass diseases [4–12], the socio-biological aspect of the epidemic is taken into account (SIR or SEIR-type compartmentalization models are used). But it is also important to hold the accompanying analysis of economic indicators which are closely interconnected with socio-biological factors. In particular, [6] emphasizes the need for quantitative assessments of the interconnection of a pandemic with economy and healthcare in order to build strategies for managing them. Therefore, a model should describe the intersection of biosocial environment, economy and healthcare, allow predicting the dynamics of mass diseases and determining optimal strategies for their containment.

1. Mathematical model

This paper presents the development of the model proposed in [13, 14].

Considering the biological aspect of the impact of a mass disease, the population (N, people) of a region is divided into the following groups:

P — persons who comply with restrictive measures (for example, self-isolation) and thereby minimize the risk of infection for themselves;

S — persons who do not comply with restrictive measures and, therefore, are potentially susceptible to infection;

E — infected persons who have the disease in the incubation stage;

I — ill persons (persons with both an asymptomatic and symptomatic forms of a disease are taken into account);

Q — ill persons, who are hospitalized;

R — recovered;

D — deceased.

The socio-economic aspect is described by the following indicators:

Y — gross output (RUB);

π — profit of an economic entity (RUB);

K — cost of fixed assets of an economic entity (RUB);

L — amount of productive labour (persons);

Z — a number of beds in hospitals to accommodate the ill (units).

In addition, investments in the implementation of the control actions of the authorities are considered:

u_1 — investments in the re-equipment of existing beds for the accommodation of the ill (RUB);

u_2 — investments in increasing the number of beds due to the construction of new hospitals (RUB);

u_3 — investments in an information campaign to combat the disease (RUB),
as well as moments of managerial decision-making:

τ_1 — a moment when governing bodies introduce restrictive measures;

τ_2 — a moment when the restrictions are lifted.

The effect of management decisions can be described as follows:

if $t = \tau_1$, then $S(t) = (1 - a)S(t)$, $P(t) = P(t) + aS(t)$;

if $t = \tau_2$, then $S(t) = S(t) + bP(t)$, $P(t) = (1 - b)P(t)$;

where a is the share of the group "persons potentially susceptible to infection", which passes into the group "persons who comply with restrictive measures" at τ_1; b is the share of the group "persons who comply with restrictive measures", which passes into the group "persons potentially susceptible to infection" at τ_2.

The mathematical model of control over an economic entity in conditions of mass diseases is formulated below as a system of differential and algebraic relations.

Dynamics of the number of persons potentially susceptible to infection:

$$\frac{dS}{dt} = k_{PS}P(t) + k_{RS}R(t - \tau) - \left(k_{SE}\left(\frac{I(t)}{N(t)}\right) + k_{SP}(U_3(t)) - \rho\right)S(t), \tag{1}$$

where ρ is natural population growth as a percentage of the total population; τ is the time during which the immunity of the recovered remains; k_{SP} is the intensity of the transition of persons at risk of infection to the group of those, who comply with restrictive measures; k_{SE} is the intensity of the transition of persons at risk of infection to the group of the infected in the incubation period; k_{RS} is the intensity of repeated morbidity; k_{PS} is the intensity with which persons drop out of the number of those who comply with restrictive measures.

In general, k_{SE} is a function of the ratio of the number of cases to the total population, and k_{SP} is a function of the amount of investment in the information campaign. Then the dynamics of the number of persons complying with restrictive measures can be represented as:

$$\frac{dP}{dt} = k_{SP}(U_3(t))S(t) - k_{PS}P(t). \tag{2}$$

Increase in the number of infected persons with the incubation stage of the disease:

$$\frac{dE}{dt} = k_{SE}\left(\frac{I(t)}{N(t)}\right)S(t) - k_{EI}E(t), \tag{3}$$

where k_{EI} is the intensity of the transition of infected persons with the incubation stage of the disease into the group of the ill.

Change in the number of cases:

$$\frac{dI}{dt} = k_{EI}E(t) - (k_{IQ} + k_{IR} + k_{ID})I(t), \tag{4}$$

where k_{IQ} is the intensity of hospitalization of the ill; k_{IR} is the intensity of recovery of non-hospitalized patients; k_{ID} is the mortality rate of non-hospitalized patients.

Dynamics of hospitalization:

$$\frac{dQ}{dt} = k_{IQ}I(t) - (k_{QD} + k_{QR})Q(t), \tag{5}$$

where k_{QD} is the mortality rate of the hospitalized ill persons; k_{QR} is the recovery rate of the hospitalized ill persons.

Increase in the number of the recovered:

$$\frac{dR}{dt} = k_{IR}I(t) + k_{QR}Q(t) - k_{RS}R(t). \tag{6}$$

Change in the number of the deceased:

$$\frac{dD}{dt} = k_{QD}Q(t) + k_{ID}I(t). \tag{7}$$

Change in the number of beds to accommodate the ill:

$$\frac{dZ}{dt} = g(u_2(t - \tilde{\tau})) - \mu Z(t) + ku_1(t), \tag{8}$$

where $g(u_2)$ is a function that matches investments in the construction of new hospitals with an increase in the number of beds; $\tilde{\tau}$ is the time of implementation of investments; μ is the depreciation of hospital funds; k is the parameter that determines the ratio of the number of beds converted to accommodate patients and investments in the conversion.

Population of a region:

$$N(t) = P(t) + S(t) + E(t) + I(t) + Q(t) + R(t). \tag{9}$$

General amount of effective labour:

$$L(t) = s_1P(t) + s_2S(t) + s_3E(t) + s_4R(t). \tag{10}$$

where $s_k = e_k \cdot m$, $k = \overline{1,4}$; m is the share of the working-age population of the total population; e_1 is the labor efficiency coefficient of a healthy person who complies with restrictive measures and works remotely; e_2 is the labor efficiency coefficient of a healthy person working full-time; e_3 is the labor efficiency coefficient of a person whose disease is in the incubation stage; e_4 is the labor efficiency coefficient of a recovered person working full-time. Thus, s_k, $k = \overline{1,4}$, is the share of investments of the productive labour of persons from the corresponding groups.

The value of the gross output of an economic entity:

$$Y(t) = F(K(t), L(t)), \tag{11}$$

where K is the cost of fixed assets of an economic entity; L is the volume of productive labour. Note that the value of $L(t)$, according to (10), takes into account the influence of a mass disease. Thus, it is indirectly contained in the value of gross output (Y).

Profit of a region:

$$\pi(t) = Y(t) - u_1(t) - u_2(t) - u_3(t). \tag{12}$$

Gross output depends on the biosocial effect of a mass disease, therefore, profit also indirectly takes this effect into account.

The volume of investments is assumed to be limited:

$$u_i(t) \geqslant 0, \quad \int_{t_0}^{T} u_i(t)dt \leqslant B_i, \quad i = \overline{1,3}, \tag{13}$$

where B_1 is the amount of the budget the conversion of existing beds; B_2 is the amount of the budget for construction of new hospitals; B_3 is the amount of the budget for an information campaign to combat mass disease.

It is assumed that the number of hospitalized does not exceed the number of beds:

$$Q(t) \leqslant Z(t). \tag{14}$$

To select a control strategy, it is necessary to introduce a criterion that allows determining the best item from a set of control strategies [15–19]. We introduce a functional that contains a combination of criteria: economic (maximizing cumulative relative profit) and social (minimizing the number of cases):

$$\int_{t_0}^{T} (\alpha_1\pi(t) - \alpha_2 E(t))dt \to \max_{u_1,u_2,u_3,\tau_1,\tau_2}, \tag{15}$$

where α_1 and α_2 are share parameters whose values should be determined by experts. The system (1)–(14) in combination with the quality criterion (15) is a problem of optimal control with a delay. Its solution requires the use of specialized numerical methods, for example, considered in [20, 21].

2. Numerical experiment

To find the optimal solution, estimates of the parameters of the model (1)–(15) are required. The graduate work by M. S. Rybina “Mathematical modeling of optimal control over an economic entity in conditions of a mass disease” (Ulyanovsk State University, 2022) provides estimates of parameters for the Russian Federation and the Ulyanovsk region based on data on the COVID-19 pandemic in 2020 (Table 1). There were also estimates of functional dependencies between the model parameters (Table 2) and based on the proposed model, the optimal control parameters for the economies of the Russian Federation and the Ulyanovsk region.

To find optimal solutions, the authors’ software package and algorithms considered in [20] were used. Parameterization method was used to find the solution of the problem (1)–(15) in the class of piecewise constant controls $u_i(t) = c_i$, $1 \leqslant i \leqslant 3$, $t \in [t_0; T]$. Here c_i are constant values, $1 \leqslant i \leqslant 3$.

Numerical experiment was held using the data obtained in the condition of absence of vaccination. The beginning of the pandemic in Russia in the middle of March 2020 was the initial point, while the final point was December 2020. Thus, the planning period considered in the problem is $t_0 = 0$, $T = 9.5$.

Tables 3 and 4 present the optimal values of the control parameters corresponding to allocated budgets. Each budget was varied by $\pm 1\%$ and $\pm 10\%$, other things being equal. The parameters of optimal solutions were compared with the corresponding parameters calculated in the absence of budget variation.

Table 1

Estimates of the parameters for the Russian Federation and the Ulyanovsk region

Parameter	Region		Parameter	Region	
	Russia	Ulyanovsk region		Russia	Ulyanovsk region
a	0.415	0.416	b	0.271	0.377
c_1	0.174	0.122	c_2	$1.845 \cdot 10^{-11}$	$2.0975 \cdot 10^{-11}$
k_{PS}	0.64	0.5	k_{QD}	0.1	0.13
k_{RS}	0.0002	0.0002	k_{IR}	2.8	2.36
k_{QR}	2.0	2.8	k_{ID}	0	0
k	$3.415 \cdot 10^{-6}$	$2.719 \cdot 10^{-6}$	m	0.482	0.467
e_1	0.879	0.879	e_2	1	1
e_3	0.43	0.43	e_4	1	1
μ	$8.33 \cdot 10^{-3}$	$8.33 \cdot 10^{-3}$	ρ	$-9.516 \cdot 10^{-5}$	$-3.636 \cdot 10^{-3}$

Table 2

Estimates of parameter functions for the Russian Federation and the Ulyanovsk region

Parameter function	Region	
	Russia	Ulyanovsk region
$k_{SE}(I)$	$1.11 \cdot 10^{-7} I$	$7.7 \cdot 10^{-6} I$
$k_{SP}(u_3)$	$0.174 + 1.845 \cdot 10^{-11} u_3$	$0.122 + 2.097 \cdot 10^{-3} u_3$
$g(u_2)$	$1.381 \cdot 10^{-7} u_2$	0
$F(K, L)$	$7.47 \cdot 10^{-5} K^{0.4387} L^{1.3667}$	$28.228 K^{0.3815} L^{0.5728}$
$K(t)$	$3.4973 \cdot 10^{14} e^{0.0102t}$	$1.256128 \cdot 10^{12} e^{0.01178t}$

Table 3

Optimal control parameters for the Russian Federation

Functional coefficients	Budget value change	u_1, RUB	u_2, RUB	u_3, RUB	τ_1, months	τ_2, months
$\alpha_1 = 1$, $\alpha_2 = 0$ (economic criterion)	–	7153989000	2521637000	43737850	0.896	2.029
	$0.9B_1$	6919106000	2712544000	47096230	0.834	2.180
	$0.9B_2$	7153989000	2280855000	43957200	0.893	2.034
	$0.9B_3$	7161150000	2526688000	39482390	0.894	2.034
	$1.1B_1$	7381289000	2367600000	41148330	0.953	1.908
	$1.1B_2$	7146835000	2776578000	43781630	0.897	2.027
	$1.1B_3$	7139688000	2524162000	48183900	0.896	2.028
	$0.99B_1$	7125093000	2544446000	44177640	0.889	2.047
	$0.99B_2$	7153989000	2498920000	43781630	0.896	2.030
	$0.99B_3$	7153989000	2526688000	43430630	0.896	2.030
	$1.01B_1$	7175102000	2506545000	43497840	0.901	2.017
	$1.01B_2$	7146835000	2549403000	43825460	0.894	2.032
	$1.01B_3$	7146835000	2529218000	44308020	0.894	2.032
$\alpha_1 = 0$, $\alpha_2 = 1$ (social criterion)	–	8463473000	3031374000	52842080	0.000	9.500
	$0.9B_1$	7617126000	3031374000	52842080	0.000	9.500
	$0.9B_2$	8463473000	2728237000	52842080	0.000	9.500
	$0.9B_3$	8463473000	3031374000	47557870	0.000	9.500

Continuation of Table 3

Functional coefficients	Budget value change	u_1, RUB	u_2, RUB	u_3, RUB	τ_1, months	τ_2, months
$\alpha_1 = 0$, $\alpha_2 = 1$ (social criterion)	$1.1B_1$	9309820000	3031374000	52842080	0.000	9.500
	$1.1B_2$	8463473000	3334511000	52842080	0.000	9.500
	$1.1B_3$	8463473000	3031374000	58126290	0.000	9.500
	$0.99B_1$	8378838000	3031374000	52842080	0.000	9.500
	$0.99B_2$	8463473000	3001060000	52842080	0.000	9.500
	$0.99B_3$	8463473000	3031374000	52313660	0.000	9.500
	$1.01B_1$	8548108000	3031374000	52842080	0.000	9.500
	$1.01B_2$	8463473000	3061688000	52842080	0.000	9.500
	$1.01B_3$	8463473000	3031374000	53370500	0.000	9.500

Table 4

Optimal control parameters for the Ulyanovsk region

Functional coefficients	Budget value change	u_1, RUB	u_3, RUB	τ_1, months	τ_2, months
$\alpha_1 = 1$, $\alpha_2 = 0$ (economic criterion)	–	1302991	43215870	1.068	1.691
	$0.9B_1$	1246425	45979050	1.003	1.795
	$0.9B_3$	1304295	38933220	1.065	1.693
	$1.1B_1$	1357908	41066070	1.126	1.604
	$1.1B_3$	1300386	47632680	1.068	1.690
	$0.99B_1$	1296430	43606770	1.060	1.702
	$0.99B_3$	1302991	42869410	1.067	1.691
	$1.01B_1$	1308145	43000220	1.073	1.682
	$1.01B_3$	1302991	43735460	1.068	1.690
$\alpha_1 = 0$, $\alpha_2 = 1$ (social criterion)	–	1578947	52842106	0.000	9.500
	$0.9B_1$	1421052	52842060	0.000	9.500
	$0.9B_3$	1578947	47557850	0.000	9.500
	$1.1B_1$	1736842	52842060	0.000	9.500
	$1.1B_3$	1578947	58126260	0.000	9.500
	$0.99B_1$	1563158	52842060	0.000	9.500
	$0.99B_3$	1578947	52313640	0.000	9.500
	$1.01B_1$	1594736	52842060	0.000	9.500
	$1.01B_3$	1578947	53370480	0.000	9.500

The analysis of the values in Tables 3 and 4 shows that:

1. The optimal solution for social and economic criteria when changing budgets is sustainable. If the budget changes by 1% (10%), the optimal parameter values change by less than 1% (10%). This means that the vast majority of the parameters of the optimal solution are weakly elastic.

2. The parameters of the optimal solution when using the economic criterion are more susceptible to change than when using the social criterion.

3. The change in the parameters of the optimal solution for both the Ulyanovsk region and the Russian Federation is common. In other words, there is a similar effect of the variables' values on the optimal strategies for economic entities of a different scale and complexity. Consequently, the applicability of the model for such subjects is confirmed.

Conclusion

The paper proposes a mathematical model as a tool for determining optimal control strategies over an economic entity in conditions of mass diseases. The model includes socio-biological and economic factors, as well as control measures.

Based on the modification of the parametrization method developed by the authors, optimal control strategies over an economy in conditions of mass disease were determined for the Russian Federation and the Ulyanovsk region. A numerical experiment aimed at studying the stability of the parameters of the optimal solution with variation in the values of budgets was held. As a result, it was shown that the parameters of the optimal solution are stable and change in a similar way for the considered economic entities. This confirms the possibility of using the model for economic entities of various scales and complexity.

Further development of the study includes supplementing the model with a vaccination factor, as well as finding and analyzing optimal solutions for non-zero values of both coefficients of the target functional.

References

1. Kul'kova I. A. The coronavirus pandemic influence on demographic processes in Russia. *Human Progress*, 2020, vol. 6, iss. 1, pp. 2–11 (in Russian). https://doi.org/10.34709/IM.161.5
2. Bobkov A. V., Vereshchagina V. K. Correctional dynamics of economic activity under the influence of measures to control the pandemic. *Innovation and Investment*, 2020, vol. 8, pp. 94–98 (in Russian). EDN: LHLLQP
3. Funk S., Gilad E., Watkins C., Jansen V. A. A. The spread of awareness and its impact on epidemic outbreaks. *Proceedings of the National Academy of Sciences*, 2009, vol. 106, iss. 16, pp. 6872–6877. https://doi.org/10.1073/pnas.0810762106
4. Wani A. U., Bakshi A., Wani M. A. Dynamics of COVID-19: Modelling and analysis. *Journal of Infectious Diseases and Epidemiology*, 2020, vol. 6, pp. 1–11. https://doi.org/10.23937/2474-3658/1510128
5. Arino J., Brauer F., van den Driessche P., Watmough J., Wu J. Simple models for containment of a pandemic. *Journal of the Royal Society Interface*, 2006, vol. 3, iss. 8, pp. 453–457. https://doi.org/10.1098/rsif.2006.0112
6. Atkeson A. What will be the economic impact of Covid-19 in the US? Rough estimates of disease scenarios. *NBER Working Papers*, 2020, Art. 26867. Available at: https://www.nber.org/system/files/working_papers/w26867/w26867.pdf (accessed May 29, 2021).
7. Brauer F., Castillo-Chavez C. *Mathematical Models in Population Biology and Epidemiology*. Texts in Applied Mathematics, vol. 40. New York, Springer, 2012. 508 p. https://doi.org/10.1007/978-1-4614-1686-9
8. Britton N. F. *Essential Mathematical Biology*. London, Springer, 2003. 335 p. https://doi.org/10.1007/978-1-4471-0049-2
9. Castillo-Chavez C., Blower S., van den Driessche P., Kirschner D., Yakubu A. A. (eds.) *Mathematical Approaches for Emerging and Reemerging Infectious Diseases: Models, Methods, and Theory*. The IMA Volumes in Mathematics and its Applications, vol. 126. New York, Springer, 2002. 377 p. https://doi.org/10.1007/978-1-4613-0065-6
10. He S., Tang S., Rong L. A discrete stochastic model of COVID-19 outbreak: Forecast and control. *Mathematical Biosciences and Engineering*, 2020, vol. 17, iss. 4, pp. 2792–2804. https://doi.org/10.3934/mbe.2020153
11. Volz E., Meyers L. A. Susceptible-infected-recovered epidemics in dynamic contact networks. *Proceedings of the Royal Society B*, 2007, vol. 274, iss. 1628, pp. 2925–2934. https://doi.org/10.1098/rspb.2007.1159

12. Matveev A. V. The mathematical modeling of the effective measures against the COVID-19 spread. *National Security and Strategic Planning*, 2020, vol. 2020, iss. 1 (29), pp. 23–39 (in Russian). https://doi.org/10.37468/2307-1400-2020-1-23-39
13. Rybina M. S. Mathematical model of optimal resource control in conditions of a pandemic. *Proceedings of the International Youth Scientific Forum "LOMONOSOV–2021"* 2021 (in Russian). Available at: https://lomonosov-msu.ru/archive/Lomonosov_2021/data/22519/127569_uid543558_report.pdf (accessed May 29, 2021).
14. Rybina M. S. The problem of estimating the parameters of the mathematical model of the impact of the pandemic on the economy. *Collection of Abstracts of the Congress of Young Scientists (ITMO University)*, 2021 (in Russian). Available at: https://kmu.itmo.ru/digests/article/7045 (accessed May 29, 2021).
15. Igret Araz S. Analysis of a Covid-19 model: Optimal control, stability and simulations. *Alexandria Engineering Journal*, 2020, vol. 60, iss. 1, pp. 647–658. https://doi.org/10.1016/j.aej.2020.09.058
16. Macalisang J., Caay M., Arcede J., Caga-Anan R. Optimal control for a COVID-19 model accounting for symptomatic and asymptomatic. *Computational and Mathematical Biophysics*, 2020, vol. 8, pp. 168–179. https://doi.org/10.1515/cmb-2020-0109
17. Ovsyannikova N. I. Problem of optimal control of epidemic in view of latent period. *Civil Aviation High Technologies*, 2017, vol. 20, iss. 2, pp. 144–152. EDN: YLJOFL
18. Zamir M., Abdeljawad T., Nadeem F., Khan A., Yousef A. An optimal control analysis of a COVID-19 model. *Alexandria Engineering Journal*, 2021, vol. 60, iss. 2, pp. 2875–2884. https://doi.org/10.1016/j.aej.2021.01.022
19. Andreeva E. A., Semykina N. A. Optimal control of the spread of an infectious disease with allowance for an incubation period. *Computational Mathematics and Mathematical Physics*, 2005, vol. 45, iss. 7, pp. 1133–1139. EDN: LJBSEB
20. Lutoshkin I. V. The parameterization method for optimizing the systems which have integro-differential equations. *The Bulletin of Irkutsk State University. Series Mathematics*, 2011, vol. 4, iss. 1, pp. 44–56 (in Russian). EDN: NQWBZH
21. Samarskiy A. A. *Vvedenie v chislennye metody* [Introduction to Numerical Methods]. St. Petersburg, Lan', 2005. 288 p. (in Russian). EDN: QJODBV

Поступила в редакцию / Received 22.11.2022
Принята к публикации / Accepted 25.12.2022
Опубликована / Published 31.05.2023