Известия Саратовского университета. Новая серия. Серия: Математика. Механика. Информатика. 2024. Т. 24, вып. 1. С. 63–70
Izvestiya of Saratov University. Mathematics. Mechanics. Informatics, 2024, vol. 24, iss. 1, pp. 63–70
mmi.sgu.ru https://doi.org/10.18500/1816-9791-2024-24-1-63-70, EDN: QACBVA

Article

Connectivity in a rough plane and axially symmetric contacts with a special coating

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Abstract. There is some evidence that in certain cases a contact of rough elastic solids is multiply connected, i.e. have regions in it where contact surfaces are apart from each other and the contact pressure is zero. The issue of the connectivity in rough elastic contacts has both theoretical and practical interest, especially for seals. In this paper, we extend the earlier conducted analysis of rough contacts without coatings in plane and axially symmetric formulations on the cases of plane and axially symmetric rough elastic contacts with special coatings and compare our findings. The main goal of the paper is to obtain the exact analytical solutions of plane and axially symmetric rough elastic contacts with a special coating and analyze their properties such as contact connectivity and contact pressure smoothness compared to the smoothness of the surface roughness profile. This goal is achieved by using solution expansions in Chebyshev and Legendre orthogonal polynomials. A range of contact parameters has been determined for which the contacts are connected individually.
Keywords: plane and axially symmetric rough coated contacts, Chebyshev and Legendre orthogonal polynomials, series convergence and solution, singly connected rough contacts
For citation: Kudish I. I. Connectivity in a rough plane and axially symmetric contacts with a special coating. *Izvestiya of Saratov University. Mathematics. Mechanics. Informatics*, 2024, vol. 24, iss. 1, pp. 63–70. https://doi.org/10.18500/1816-9791-2024-24-1-63-70, EDN: QACBVA
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Научная статья
УДК 539.3

Соединение в шероховатой плоскости и осесимметричных контактах со специальным покрытием

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Аннотация. Имеются некоторые свидетельства того, что в определенных случаях контакт шероховатых упругих тел является многосвязным, т. е. в нем имеются области, где контактные поверхности находятся на расстоянии друг от друга, а контактное давление равно нулю. Вопрос о соединении в шероховатых упругих контактах представляет как теоретический, так и практический интерес, особенно для уплотнений. В этой статье мы расширяем ранее проведенный анализ шероховатых контактов без покрытий в плоских и осесимметричных композициях на случаи плоских и осесимметричных шероховатых упругих контактов со специальными покрытиями и сравниваем наши результаты. Основная цель работы — получить точные аналитические решения для плоских и осесимметричных

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шероховатых упругих контактов со специальным покрытием и проанализировать их свойства, такие как контактная связность и гладкость контактного давления, по сравнению с гладостью профиля шероховатости поверхности. Эта цель достигается за счет использования разложений решения в ортогональные многочлены Чебышева и Лежандра. Был определен диапазон контактных параметров, для которых контакты односвязны.

Ключевые слова: плоские и осесимметричные контакты с шероховатым покрытием, ортогональные многочлены Чебышева и Лежандра, сходимость рядов и решение, односвязные контакты с шероховатым покрытием

Для цитирования: *Kudish I. I.* Connectivity in a rough plane and axially symmetric contacts with a special coating [*Кудиш И. И.* Соединение в шероховатой плоскости и осесимметричных контактах со специальным покрытием] // Известия Саратовского университета. Новая серия. Серия: Математика. Механика. Информатика. 2024. Т. 24, вып. 1. С. 63–70. https://doi.org/10.18500/1816-9791-2024-24-1-63-70, EDN: QACBVA

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Introduction

The studies of elastic rough contacts have a long history. A number of analytical and semi-analytical models using various assumptions (such as wavy surfaces, partial contact, periodic and random surface profiles) have been proposed [1–6], including models with adhesion [7–9] and numerical studies [10]. In [11], the authors analyzed analytically rough contacts with roughness described by the Weierstrass function, which is continuous everywhere but not differentiable anywhere. This analysis led to the conclusion that, under such an assumption, a rough contact is always multi-connected, i.e. it has a series of places where contact pressure is positive (actual contact takes place) and a series of areas where contact surfaces are apart from each other, i.e. pressure is zero. In all these studies, the elastic solids were assumed to be made of a homogeneous elastic material, and no coatings of any kind were considered.

That brought to the forefront two interconnected questions about solids made of homogeneous elastic material and elastic solids with coatings: (a) What would be a reasonable assumption about the smoothness of rough surfaces? and (b) Is it possible to get singly connected rough contacts? In [12–14], we see the first attempts to answer both of these questions for contacts without coatings and show how they are interconnected. In [12], besides theoretical analysis, some experimental studies of real ground surfaces using optical and electron force microscopes were performed. The connectivity of real rough surfaces has certain serious practical consequences such as leakage through the gaps between surface asperities in contact sealing lubricated spaces. Therefore, besides a purely theoretical interest in the issue of rough contact connectivity, there is also a practical one.

The main goal of this paper is to extend the findings of [12–14] on the cases of plane and axially symmetric rough contacts with special coatings. In other words, it is to analyze the connectivity in a contact of rough surface and to analyze the dependence on the smoothness of the roughness profile and connectivity. In the process, the exact solutions of the plane and axially symmetric problems for rough elastic contact with special coatings with the help of Chebyshev and Legendre orthogonal polynomials.

1. Formulation of the problem for rough plane contacts with a coating

Let us assume that a rigid infinite in the y direction punch with the bottom half-width a is indented in a coated half-plane made of a homogeneous elastic material with elastic modulus E and Poisson's ratio ν (see Figure).

Let us assume that the coating is thin and its vertical displacement is represented by the Winkler – Fuss relationship with the coefficient proportionality dependent on the distance from the contact center, i.e. $w_c(x) = \lambda\sqrt{1-x^2/a^2}p(x)$, where $w_c(x)$ is the vertical displacement of the thin coating subjected to pressure $p(x)$, x is the coordinate of the point in the contact, λ is a constant nonnegative coefficient proportionality. The coordinate system is introduced in such a way that the z-axis is directed upward, the y-axis is directed along the punch length, and the x-axis is directed along the punch contact with the half-plane. The punch bottom texture is described by a continuous function $z = f(x)$. It is assumed that the contact is frictionless. The load applied to the punch is directed along the negative z-axis and is equal to P. In this classical formulation for singly connected contacts the problem equations are as follows [15]

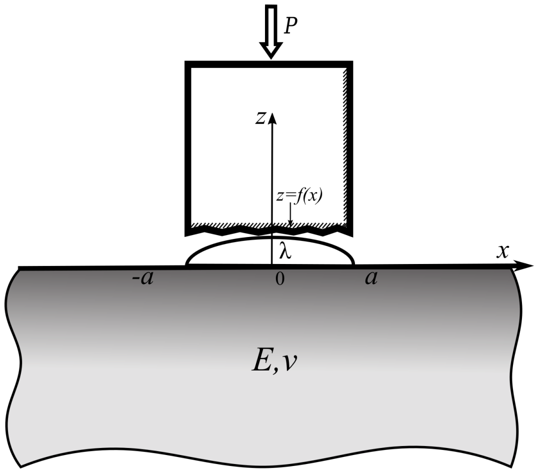

Figure. The general view of a rigid punch indented in an elastic half-plane with a special coating

$$\lambda\sqrt{1-x^2/a^2}p(x) + \frac{2(1-\nu^2)}{\pi E}\int\limits_{-a}^{a} p(t)\ln\frac{a}{\mid x-t\mid}dt = \delta - f(x), \quad \int\limits_{-a}^{a} p(x)dx = P, \tag{1}$$

where the contact pressure $p(x)$ and the rigid vertical displacement of the punch δ caused by the applied load P are unknown and need to be determined.

2. Analysis of the plane problem

Let us use for solution of the formulated problem expansions in series with respect to the Chebyshev orthogonal polynomials $T_n(x)$ of the first kind [16]. Specifically, let us assume that

$$f(x) = \omega f_0(x), \ f_0(x) = \sum_{n=0}^{\infty}\alpha_n T_n\left(\frac{x}{a}\right),$$
$$\alpha_0 = \frac{1}{\pi}\int\limits_{-a}^{a}\frac{f_0(ax)dx}{\sqrt{1-x^2}}, \quad \alpha_n = \frac{1}{\pi}\int\limits_{-a}^{a}\frac{f_0(ax)T_n(x)dx}{\sqrt{1-x^2}}, \quad n = 1,2,\ldots, \tag{2}$$

where coefficients α_n are known. Here ω is a dimensionless constant characterizing the overall height of the asperity profile described by function $f(x)$ while $f_0(x)$ describes the nominal roughness profile.

We will need to use the following relationships [17]

$$\frac{1}{\pi}\int\limits_{-1}^{1}\ln\frac{1}{\mid x-y\mid}\frac{dy}{\sqrt{1-y^2}} = \ln 2, \quad \frac{1}{\pi}\int\limits_{-1}^{1}\ln\frac{1}{\mid x-y\mid}\frac{T_n(y)dy}{\sqrt{1-y^2}} = \frac{1}{n}T_n(x), \quad n = 1,2,\ldots \tag{3}$$

The solution to Problem (1) will be searched in the form of series, i.e.

$$p(x) = \sum_{n=0}^{\infty}\beta_n\frac{T_n\left(\frac{x}{a}\right)}{\sqrt{1-x^2/a^2}}, \tag{4}$$

where coefficients β_n, $n = 0,1,\ldots$, are unknown and need to be determined. Substituting (4) into equations (1) and taking into account the orthogonality of $T_n(x)$, $n = 0,1,\ldots$, on interval $-1 < x < 1$ one obtains

$$\lambda\sum_{n=0}^{\infty}\beta_n T_n\left(\frac{x}{a}\right) + \frac{2(1-\nu^2)}{E}a\left\{\beta_0\ln 2 + \sum_{n=1}^{\infty}\frac{\beta_n}{n}T_n\left(\frac{x}{a}\right)\right\} =$$

$$= \delta - \omega \sum_{n=0}^{\infty} \alpha_n T_n \left(\frac{x}{a}\right), \quad | x |< a, \quad \beta_0 = \frac{P}{\pi a}. \tag{5}$$

From these equations, it is easy to find that

$$\delta = \omega\alpha_0 + \frac{P}{\pi a}\left[\lambda + 2\ln 2\frac{1-\nu^2}{E}a\right], \quad \alpha_{n0} = \left[\lambda + \frac{2(1-\nu^2)}{E}\frac{a}{n}\right]^{-1},$$
$$p(x) = \frac{1}{\sqrt{1-x^2/a^2}}\left\{\frac{P}{\pi a} - \omega\sum_{n=1}^{\infty}\alpha_n\alpha_{n0}T_n\left(\frac{x}{a}\right)\right\}, \ | x |< a. \tag{6}$$

Due to the fact that for $\lambda > 0$ the series $\sum\limits_{n=1}^{\infty}\alpha_n\alpha_{n0}$ converges absolutely while the series $\sum\limits_{n=1}^{\infty}\alpha_n\alpha_{n0}T_n\left(\frac{x}{a}\right)$ converges absolutely and uniformly (remember that $| T_n\left(\frac{x}{a}\right) |\leqslant 1$ for $| x |\leqslant a$) if the series $\sum\limits_{n=1}^{\infty}\alpha_n$ converges absolutely. For the latter to take place it is sufficient to assume that

$$\alpha_n = O\left(\frac{1}{n^\gamma}\right), \quad \gamma > 1, \quad n \to \infty. \tag{7}$$

Moreover, the higher the value of γ the more differentiable are functions $f(x)$ and $p(x)$ (see [12]).

For the pressure function $p(x)$ from (6) to be nonnegative in the entire contact region it is sufficient for the following inequality

$$\frac{P}{\pi a} - \omega\sum_{n=1}^{\infty}\alpha_n\alpha_{n0}T_n\left(\frac{x}{a}\right) \geqslant 0, \quad | x |< a, \tag{8}$$

to be valid. Due to the fact that $| T_n(x) |\leqslant 1$, $n = 0, 1, \ldots$, for $| x |\leqslant 1$ for this inequality to be true it is sufficient that

$$\frac{P}{\pi a} - \omega\sum_{n=1}^{\infty} | \alpha_n | \alpha_{n0} \geqslant 0. \tag{9}$$

Therefore, due to the convergence of the series in (9) there exists a finite positive number ω_0 that for any ω from the interval

$$0 \leqslant \omega \leqslant \omega_0 = \frac{P}{\pi a}\left\{\sum_{n=1}^{\infty} | \alpha_n | \alpha_{n0}\right\}^{-1}, \tag{10}$$

the distribution of contact pressure $p(x)$ is nonnegative in the entire contact region. For the strict positivity of $p(x)$ it is sufficient to require that $0 \leqslant \omega < \omega_0$.

Using the exact solutions (6) and (10) it is easy to calculate contact pressure $p(x)$ and the range of the parameter ω for which the contact pressure is nonnegative in the contact region. As it was shown in [12–14] by a series of measurements of real ground (rough) steel surfaces on an optical profiler profiler and an electronic force microscope, theoretically, the real rough surfaces are described by not just continuous but smooth distribution functions. The differentiability of pressure $p(x)$ from (6) for $\lambda > 0$, $0 \leqslant \omega \leqslant \omega_0$, and $| x |< a$ is the same as the differentiability of the roughness distribution $f(x)$ for $| x |< a$ as the convergence of the series $\sum\limits_{n=1}^{\infty} | \alpha_n | \alpha_{n0}$ and the series $\sum\limits_{n=1}^{\infty} | \alpha_n |$ for $f(x)$ are the same. The case of $\lambda = 0$ is analyzed in [12].

It is interesting to note that the behavior of $p(x)$ near the contact boundaries $x = \pm a$ in the present problem is very different from the behavior of $p(x)$ in the plane contact problem with a coating which normal displacement $w_c(x)$ subjected to pressure $p(x)$ is described by the relationship $w_c(x) = \lambda p(x)$, where λ is a constant nonnegative coefficient proportionality [15]. Specifically, in the present problem $p(x) \to +\infty$ as $r \to \pm a$ while in [15] $p(x) \to constant$ as $r \to \pm a$. This behavior of $p(x)$ in our problem becomes obvious from the asymptotically valid solution $p(x) = \frac{1}{\lambda}\frac{\delta - f(x)}{\sqrt{1-x^2/a^2}} + \dots$ for $\lambda \gg 1$.

3. Formulation of the problem for rough axially symmetric contacts with a coating

Let us consider an axially symmetric rigid punch of radius a which is indented in a coated half-space made of a homogeneous elastic material with elastic modulus E and Poisson's ratio ν. Let us assume that the coating is thin and its vertical displacement is represented by the Winkler-Fuss relationship with the coefficient proportionality dependent on the distance from the contact center, i.e. $w_c(r) = \lambda\sqrt{1 - r^2/a^2}p(r)$, where $w_c(r)$ is the vertical displacement of the thin coating subjected to pressure $p(r)$, r is the radial distance from the contact center, λ is a constant nonnegative coefficient proportionality. The coordinate system is introduced in such a way that the z-axis is directed upward along the punch axis while in the xy-plane a polar coordinate system with radial variable $r = \sqrt{x^2 + y^2}$ is introduced. The contact arrangement is similar to the one shown in Figure. The punch bottom texture is described by a continuous function $z = f(r)$. It is assumed that the contact is frictionless. The load applied to the punch is directed along the negative z-axis and is equal to P. In this classical formulation for singly connected contacts the problem equations are as follows [15]

$$\lambda\sqrt{1 - \frac{r^2}{a^2}}p(r) + \frac{8(1-\nu^2)}{\pi E}\int\limits_0^a \frac{\rho}{r+\rho}K\left(\frac{2\sqrt{r\rho}}{r+\rho}\right)p(\rho)d\rho = \delta - f(r), \quad \int\limits_0^a rp(r)dr = \frac{P}{2\pi}, \quad (11)$$

where $K(\cdot)$ is the full elliptic integral of the second kind [16], $p(r)$ and δ are the unknown pressure and punch rigid displacement which need to be determined.

4. Analysis of the axially symmetric problem

First, let us notice that the set of all Legendre orthogonal polynomials $P_{2n}\left(\sqrt{1 - \frac{r^2}{a^2}}\right)$, $n = 0, 1, \dots$, is complete in the functional space $L_2(0, 1)$ of all quadratically integrable functions on the interval $(0, 1)$. The same set of functions represents the basis in the functional space $C(0, 1)$ of all continuous functions on $(0, 1)$. Therefore, any continuous function $f(r)$ describing the the texture of the punch bottom can be represented in the form [16]

$$f(r) = \omega f_0(r), \ \ f_0(r) = \sum_{n=0}^{\infty}\alpha_n P_{2n}\left(\sqrt{1 - \frac{r^2}{a^2}}\right),$$
$$\alpha_n = (4n+1)\int\limits_0^1 f_0(ar)P_{2n}\left(\sqrt{1-r^2}\right)\frac{rdr}{\sqrt{1-r^2}}, \quad (12)$$

where coefficients α_n are known. Here ω is a dimensionless constant characterizing the overall height of the asperity profile described by function $f(r)$ while $f_0(r)$ describes the nominal roughness profile.

Due to the convergence of this series in $L_2(0, 1)$ it also converges to our continuous function $f(r)$ almost everywhere (possibly, except for a set of points from $(0, 1)$ of measure zero). On the other hand, the series for $f_0(r)$ in (12) is a power series and, therefore, within any closed region $[b, c] \subset (0, 1)$ it converges absolutely and uniformly. It means that this series not only converges to a continuous function $f_0(r)$ in $(0, 1)$ but it is also a differentiable function.

We are ready to find the solution to our problem (11) using the following relationship [17]

$$\int_0^1 \frac{\rho P_{2n}\left(\sqrt{1-\rho^2}\right)}{(r+\rho)\sqrt{1-\rho^2}} K\left(\frac{2\sqrt{r\rho}}{r+\rho}\right) d\rho = \left\{\frac{\pi}{2}\frac{(2n-1)!!}{(2n)!!}\right\}^2 P_{2n}\left(\sqrt{1-r^2}\right). \tag{13}$$

Let us search for the solution of problem (11) in the form

$$p(r) = \sum_{n=0}^{\infty} \beta_n \frac{P_{2n}\left(\sqrt{1-\frac{r^2}{a^2}}\right)}{\sqrt{1-\frac{r^2}{a^2}}}, \tag{14}$$

where constants β_n are unknown and have to be determined from the solution.

Substituting (12) and (14) into (11) and using (13) we obtain

$$\lambda \sum_{n=0}^{\infty} \beta_n P_{2n}\left(\sqrt{1-\frac{r^2}{a^2}}\right) + \frac{8(1-\nu^2)}{\pi E} a \sum_{n=0}^{\infty} \beta_n \left\{\frac{\pi}{2}\frac{(2n-1)!!}{(2n)!!}\right\}^2 P_{2n}\left(\sqrt{1-\frac{r^2}{a^2}}\right) =$$
$$= \delta - \omega \sum_{n=0}^{\infty} \alpha_n P_{2n}\left(\sqrt{1-\frac{r^2}{a^2}}\right), \quad \sum_{n=0}^{\infty} \beta_n \int_0^a r \frac{P_{2n}\left(\sqrt{1-\frac{r^2}{a^2}}\right)}{\sqrt{1-\frac{r^2}{a^2}}} dr = \frac{P}{2\pi}. \tag{15}$$

Here, we interchanged the order of integration and summation which is legitimate for the series in (12) being convergent uniformly in any closed interval $[b, c] \subset (0, a)$.

Using the orthogonality of polynomials P_{2n} from (14) we find

$$\delta = \omega\alpha_0 + \frac{P}{2\pi a^2}\left(\lambda + 2\pi a \frac{1-\nu^2}{E}\right),$$
$$\beta_0 = \frac{P}{2\pi a^2}, \quad \beta_n = -\frac{\omega\alpha_n}{\lambda + 2\pi a \frac{1-\nu^2}{E}\left\{\frac{(2n-1)!!}{(2n)!!}\right\}^2}, \quad n = 1, 2, \dots \tag{16}$$

Therefore, based on (14) and (16) the solution to our problem (11) has the form

$$\delta = \omega\alpha_0 + \frac{P}{2\pi a^2}\left(\lambda + 2\pi a \frac{1-\nu^2}{E}\right), \quad \beta_{n0} = \left\{\lambda + 2\pi a \frac{1-\nu^2}{E}\left[\frac{(2n-1)!!}{(2n)!!}\right]^2\right\}^{-1},$$
$$p(r) = \frac{1}{\sqrt{1-\frac{r^2}{a^2}}}\left\{\frac{P}{2\pi a^2} - \omega \sum_{n=1}^{\infty} \alpha_n \beta_{n0} P_{2n}\left(\sqrt{1-\frac{r^2}{a^2}}\right)\right\}. \tag{17}$$

Obviously, for $\lambda > 0$ one has

$$\mid \beta_n \mid < \frac{\omega}{\lambda} \mid \alpha_n \mid, \quad n = 1, 2, \dots \tag{18}$$

Therefore, if the series $\sum\limits_{n=1}^{\infty} \mid \alpha_n \mid$ converges then for $\lambda > 0$ the series $\sum\limits_{n=1}^{\infty} \alpha_n \beta_{n0}$ converges absolutely and series $\sum\limits_{n=1}^{\infty} \alpha_n \beta_{n0} P_{2n}\left(\sqrt{1-\frac{r^2}{a^2}}\right)$ converges absolutely and uniformly for $0 \leqslant r \leqslant a$ because $\mid P_{2n}\left(\sqrt{1-\frac{r^2}{a^2}}\right) \mid \leqslant 1$ for $0 \leqslant r \leqslant a$.

Solution (17) indicates that pressure $p(r)$ is nonnegative if

$$\frac{P}{2\pi a^2} - \omega \sum_{n=1}^{\infty} \alpha_n \beta_{n0} P_{2n}\left(\sqrt{1-\frac{r^2}{a^2}}\right) \geqslant 0, \quad 0 \leqslant r \leqslant a. \tag{19}$$

Taking into account the fact that $\mid P_{2n}(x) \mid \leqslant 1$ for all $0 \leqslant x \leqslant 1$ [16] it becomes clear that it is sufficient for parameter ω to be small enough for the inequality

$$\frac{P}{2\pi a^2} - \omega \sum_{n=1}^{\infty} \mid \alpha_n \mid \beta_{n0} \geqslant 0. \tag{20}$$

to be valid. The latter always takes place if the series $\sum\limits_{n=1}^{\infty} \mid \alpha_n \mid$ is convergent. Therefore, if (20) is satisfied then the function of pressure $p(r)$ is nonnegative in $[0, a)$, and the contact is singly connected. If in (20) we have a strict inequality then $p(r)$ is positive in $[0, a)$.

For $\lambda > 0$ the above conclusions are certainly true if α_n satisfy (7). By the way, inequality (7) guarantees that the series in (12) converges uniformly for $0 \leqslant r \leqslant a$ and, therefore converges everywhere in this interval to a continuous function $f(r)$. In other words, if α_n satisfy (7) then there exists such a finite positive number ω_0 that for

$$0 \leqslant \omega < \omega_0 = \frac{P}{2\pi a^2} / \sum_{n=1}^{\infty} \mid \alpha_n \mid \beta_{n0}, \quad \beta_{n0} = \left\{ \lambda + 2\pi a \frac{1-\nu^2}{E} \left[\frac{(2n-1)!!}{(2n)!!} \right]^2 \right\}^{-1}, \tag{21}$$

the function of pressure $p(r)$ is positive in the entire contact region $0 \leqslant r < a$.

Using the exact solution (17) and (21) it is easy to calculate contact pressure $p(r)$ and the range of the parameter ω for which the contact pressure is nonnegative in the contact region. The differentiability of pressure $p(r)$ from (17) for $\lambda > 0$, $0 \leqslant \omega < \omega_0$, and $0 \leqslant r < a$ is the same as the differentiability of the roughness distribution $f(r)$ due to the fact that the series $\sum\limits_{n=1}^{\infty} \mid \alpha_n \mid \beta_{n0}$ converges exactly the same way as the series $\sum\limits_{n=1}^{\infty} \mid \alpha_n \mid$ of the absolute values of the coefficients α_n of the series for $f(r)$. With respect to the differentiability of real ground surfaces, please see [12]. The case of $\lambda = 0$ is analyzed in [14].

It is interesting to note that the behavior of $p(r)$ near the contact boundary in the present problem is very different from the behavior of $p(r)$ in the axially symmetric contact problem with a coating which normal displacement $w_c(r)$ subjected to pressure $p(r)$ is described by the relationship $w_c(r) = \lambda p(r)$, where λ is a constant nonnegative coefficient proportionality [15]. Specifically, in the present problem $p(r) \to +\infty$ as $r \to a$ while in [15] $p(r) \to constant$ as $r \to a$. This behavior of $p(r)$ in our problem becomes obvious from the asymptotically valid solution $p(r) = \frac{1}{\lambda} \frac{\delta - f(r)}{\sqrt{1 - r^2/a^2}} + \ldots$ for $\lambda \gg 1$.

Closure

A quantitative and qualitative analysis of plane and axially symmetric frictionless contacts with special coatings has been proposed. The exact solutions of the problems in series are obtained. In both plane and axially symmetric cases of rough elastic contacts with coatings the ranges of contact parameters for which contacts are singly connected have been determined. The limit of the overall roughness height ω_0 below which it is guaranteed that the pressure distribution is positive within the entire contact depends on the material elastic parameters, coating property, applied load, contact size, and the nominal roughness distribution. It is clear that as the elastic modulus E and applied load P increase while the coating coefficient λ and contact size a decrease the range of the surface roughness $[0, \omega_0]$ for which a singly connected contact is possible increases.

The solutions obtained above for the cases of the plane and axially symmetric problem formulations with fixed contact boundaries can be used for the solution of contact problems with unknown contact boundaries (see [13]).

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Поступила в редакцию / Received 7.12.2023
Принята к публикации / Accepted 28.12.2023
Опубликована / Published 01.03.2024