Известия Саратовского университета. Новая серия. Серия: Математика. Механика. Информатика. 2026. Т. 26, вып. 1. С. 132–138
Izvestiya of Saratov University. Mathematics. Mechanics. Informatics, 2026, vol. 26, iss. 1, pp. 132–138
https://mmi.sgu.ru DOI: https://doi.org/10.18500/1816-9791-2026-26-1-132-138
EDN: https://elibrary.ru/WNIFEO

Article

European option pricing on an incomplete market as an antagonistic game

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Abstract. We describe in detail the stochastic multi-step game corresponding to the European option pricing problem on an incomplete market with discrete time and a finite number of assets, without transaction costs and trading restrictions. Recurrent Bellman-type relations for the upper and lower guaranteed values of the game are given. The equivalence of the following statements is established: the market model is arbitrage-free; there are option seller's portfolios delivering minimum in the Bellman-type relations; there is a super-hedging portfolio for the European option. The game with an arbitrage-free market model results in an equilibrium. Based on this statement, we propose to construct a super-hedging portfolio via a seller's game strategy. Examples of analytical European option pricing on an incomplete market with a finite support and numerical pricing of an option for gold are given.
Keywords: European option, incomplete market, superhedging, antagonistic game, game equilibrium
For citation: Zverev O. V., Shelemekh E. A. European option pricing on an incomplete market as an antagonistic game. *Izvestiya of Saratov University. Mathematics. Mechanics. Informatics*, 2026, vol. 26, iss. 1, pp. 132–138. DOI: https://doi.org/10.18500/1816-9791-2026-26-1-132-138, EDN: WNIFEO

Научная статья
УДК 519.86

Расчет европейского опциона на неполном рынке как антагонистическая игра

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Аннотация. В статье подробно описана стохастическая многошаговая игровая постановка задачи расчета европейского опциона на неполном рынке с дискретным временем и конечным числом активов, без транзакционных издержек и торговых ограничений. Приведены рекуррентные соотношения беллмановского типа для верхнего и нижнего гарантированных значений игры. Установлена эквивалентность следующих утверждений: рынок безарбитражен; существуют портфели продавца опциона, доставляющие экстремум в уравнениях беллмановского типа; найдется суперхеджирующий портфель европейского опциона. Показано, что на безарбитражном рынке всегда имеет место игровое равновесие. Предложен способ построения суперхеджирующего портфеля как стратегии продавца из описанной игры. Приведены примеры аналитического расчета европейского опциона на неполном рынке с конечным носителем и числового расчета опциона на золото по данным биржи.

Ключевые слова: европейский опцион, неполный рынок, суперхеджирование, антагонистическая игра, игровое равновесие

Для цитирования: *Zverev O. V., Shelemekh E. A.* European option pricing on an incomplete market as an antagonistic game [*Зверев О. В., Шелемех Е. А.* Расчет европейского опциона на неполном рынке как антагонистическая игра] // Известия Саратовского университета. Новая серия. Серия: Математика. Механика. Информатика. 2026. Т. 26, вып. 1. С. 132–138. DOI: https://doi.org/10.18500/1816-9791-2026-26-1-132-138, EDN: WNIFEO

Introduction

Superhedging is one of the most well-known methods to hedge obligations resulting from the sale of a contingent claim on an incomplete market. The idea of superhedging is to find a self-financing trading portfolio with minimal initial investment which a.s. covers any possible future obligations of the option seller [1, p. 394]. The existence conditions for such a portfolio in the case of discrete-time market models were first established with the use of the optional decomposition technique by H. Föllmer and Y. Kabanov [2]. Similar results for continuous-time market models are obtained in [3] (for the case of risky asset price dynamics defined by a diffusion process) and in [4] (for the general semimartingale setting). However, there remains an issue with the superhedging portfolios' construction.

In search of a construction method, S. N. Smirnov and V. M. Khametov independently proposed a game setting for the option pricing problem on an incomplete market ([5, 6], respectively). S. N. Smirnov calculates an option within the guaranteed deterministic framework developed by him. The main results are collected in [7].

Here, we develop the approach proposed by V. M. Khametov. The formulation and solution methods for the game were proposed by him. Implementation is ours. The essence is as follows. In a discrete-time incomplete market model with a finite number of assets, the European option pricing problem is formulated as a multi-step stochastic antagonistic game. The players are the option seller and the market. The market opts for a probability distribution of risky asset prices equivalent to the basic one. The option seller manages a self-financing portfolio formed initially due to the sale of the contract. By assumption, there are no transaction costs and no trade restrictions. The game implies two stochastic optimization problems: the minimax and the maximin ones. The minimax problem is a well-studied one [8, 9]. At the same time, the maximin problem has not yet been studied enough; one does not know if there is an equilibrium and a saddle point in the game. The article presents the existence conditions for the seller's minimax and maximin strategies and for a game equilibrium, and explains the relationship between the game and the superhedging problem. At the end of the article, examples of analytical calculation for an option with a convex payoff function on a market with a finite support and corresponding numerical results for a vanilla call for gold are given.

1. The game

Suppose there are:

1) a probability space $(\Omega, \mathcal{F}, \mathsf{P})$, where N is a positive integer (horizon), $\Omega = (\mathbb{R}^{+})^{d(N+1)}$, $\mathcal{F} := \mathcal{F}_N := \mathcal{B}(\Omega)$, $N_0 := \{0, 1, 2, ..., N\}$ and $N_1 = N_0 \setminus \{0\}$;

2) d-dimensional random variables $\{S_t\}_{t \in N_0}$ on it.

Let us define $\mathcal{F}_0 := \{\varnothing, \Omega\}$, $\mathcal{F}_t = \sigma(S_s, s \leqslant t)$, $t \in N_1$. To simplify the recording, we assume that $\mathcal{F}_{-1} := \mathcal{F}_0$. Without loss of generality, let us consider filtration $(\mathcal{F}_t)_{t\in N_0}$ as a complete one. The introduced objects define a multistep stochastic model of a financial market with discrete time and finite horizon N, consisting of d risky assets with price evolution described by random variables $\{S_t\}_{t\in N_0}$ and one risk-free asset with constant price equal to 1 [1, Section 5.1]. Moments $t \in N_1$ are the moments of bidding. For brevity, we write $S_{u,v} := (S_u, ..., S_v)$, $0 \leqslant u < v \leqslant N$.

Suppose there is also a European contingent claim f_N [1, Definition 5.20]. The latter means that f_N is an integrable a.s. nonnegative $\mathcal{F}_N$-measurable random variable. Throughout, we will assume f_N to be a.s. bounded from above with some constant $c_1 > 0$.

Let us define *the strategies set for the market* as the set $\mathfrak{R}$, consisting of P and all probability measures Q defined on $(\Omega, \mathcal{F})$ and equivalent to P. The option seller manages a self-financing portfolio [1, Definition 5.3] formed initially due to the sale of the contract. By assumption, there are no transaction costs and no trade restrictions. The number of asset units in a portfolio at the moment $t \in N_0$ is modeled by $\mathcal{F}_{t-1}$-measurable random variables: a one-dimensional β_t for the risk-free asset and a d-dimensional γ_t for the risky assets. We denote $\beta_{u,v} := \{\beta_t\}_{u\leqslant t\leqslant v}$ and $\gamma_{u,v} := \{\gamma_t\}_{u\leqslant t\leqslant v}$, $0 \leqslant u < v \leqslant N$. Portfolio is a pair $\pi_{0,N} = \{\beta_t, \gamma_t\}_{t\in N_0}$. It is well known [1, Remark 5.8], that a self-financing portfolio is fully specified by the number of units for all assets in it at the time moment $t = 0$ and the number of risky assets units in it at any $t \in N_1$. The total initial value of assets in the portfolio is determined by the value of the option premium. Due to the absence of transaction costs, one may distribute this value between assets in any possible way. Thus, it is sufficient to consider $\gamma_{1,N}$ as an *option seller's strategy*. By U we denote the set of all $\gamma_{1,N}$, consisting of d-dimensional a.s. finite $\mathcal{F}_{t-1}$-measurable random variables γ_t, $t \in N_1$. $U_{u,s}$ denotes narrowing of U for $\{u, ..., v\}$, $1 \leqslant u < v \leqslant N$, $\gamma_{u,v} := \{\gamma_t\}_{u\leqslant t\leqslant v}$ are elements of $U_{u,s}$. If $u = s$, we write γ_s and U_s. The value of a self-financing portfolio π at the moment t is denoted by X_t^π, $t \in N_1$. One has $X_t^\pi = X_0^\pi + G_{1,t}^\gamma$ [1, Proposition 5.7], where $G_{u,t}^\gamma := \sum\limits_{s=u}^{t}\sum\limits_{i=1}^{d} \gamma_s^{(i)} \triangle S_s^{(i)}$, $1 \leqslant u \leqslant t \leqslant N$, and $\triangle S_t^{(i)} := S_t^{(i)} - S_{t-1}^{(i)}$. The random variable $G_{u,t}^\gamma$ is called the gains process for the investment period from u to t associated with a strategy $\gamma_{u,t}^N$ [1, Definition 5.6].

By the portfolio with consumption, we mean the pair (π, C) with a self-financing portfolio π and an adapted set of a.s. non-negative random variables $\{C_t\}_{t\in N_0}$ called consumption. The value of a portfolio with consumption (π, C) is defined by the formula $X_t^{(\pi,C)} := X_t^\pi - C_t$.

We will also assume that:

1) all information $\mathcal{F}_t$ is available to both players at any given time moment $t \in N_0$;

2) players act independently of each other;

3) if the players have chosen strategies $\mathsf{Q} \in \mathfrak{R}$ and $\gamma_{1,N} \in U$, then *the option seller's risk* is determined by the formula

$$I_0^{\mathsf{Q},\gamma} := I_0^{\mathsf{Q},\gamma_{1,N}}(S_0) := \mathsf{E}^{\mathsf{Q}}\left[\exp\left\{f_N - G_{1,N}^\gamma\right\}\right],$$

and *the gain of the market* is equal to $-I_0^{\mathsf{Q},\gamma}$.

Thus, we consider the dynamic stochastic antagonistic game $\Gamma := \left(\mathfrak{R}, U, I_0^{\mathsf{Q},\gamma}\right)$. They say, there is an equilibrium in the game Γ if $\inf\limits_{\gamma\in U}\sup\limits_{\mathsf{Q}\in\mathfrak{R}} I_0^{\mathsf{Q},\gamma} = \sup\limits_{\mathsf{Q}\in\mathfrak{R}}\inf\limits_{\gamma\in U} I_0^{\mathsf{Q},\gamma}$. So, we have to consider two problems: 1) the minimax one, i.e., to find $\inf\limits_{\gamma\in U}\sup\limits_{\mathsf{Q}\in\mathfrak{R}} I_0^{\mathsf{Q},\gamma}$ and 2) the maximin one, i.e., to find $\sup\limits_{\mathsf{Q}\in\mathfrak{R}}\inf\limits_{\gamma\in U} I_0^{\mathsf{Q},\gamma}$. The problems are stated correctly. Indeed, by assumption, f_N is a.s. bounded, there is no restriction on the "do not invest in risky assets" strategy. So, one has the inequalities:

$$0 < \sup_{\mathsf{Q}\in\mathfrak{R}}\inf_{\gamma\in U} I_0^{\mathsf{Q},\gamma} \leqslant \inf_{\gamma\in U}\sup_{\mathsf{Q}\in\mathfrak{R}} I_0^{\mathsf{Q},\gamma} \leqslant \sup_{\mathsf{Q}\in\mathfrak{R}} \mathsf{E}^{\mathsf{Q}}\left[\exp\{f_N\}\right] \leqslant e^{c_1}. \tag{1}$$

2. Key results

We will try a stochastic variant of the dynamic programming method to solve the problems. To justify our choice of method, let us obtain recurrent relations for the upper and lower guaranteed values of the game first. We will need the following designations, $t \in N_1$:

1) $I_{t-1}^{\mathsf{Q},\gamma} = I_{t-1}^{\mathsf{Q},\gamma_{t,N}}(S_{0,t-1}) := \mathsf{E}^{\mathsf{Q}}\left[\exp\left\{f_N - G_{t,N}^{\gamma}\right\}|\mathcal{F}_{t-1}\right]$, $\mathsf{Q} \in \mathfrak{R}$, $\gamma_{t,N} \in U_{t,N}$;

2) upper (lower) guaranteed value

$$\hat{V}_{t-1} := \operatorname*{ess\,inf}_{\gamma \in U_{t,N}} \operatorname*{ess\,sup}_{\mathsf{Q}\in\mathfrak{R}} I_{t-1}^{\mathsf{Q},\gamma}, \quad \left(\check{V}_{t-1} := \operatorname*{ess\,sup}_{\mathsf{Q}\in\mathfrak{R}} \operatorname*{ess\,inf}_{\gamma \in U_{t,N}} I_{t-1}^{\mathsf{Q},\gamma}\right).$$

Random variables $\hat{V}_t$ and $\check{V}_t$ are a.s. bounded. One may prove it in the same way as it was done for inequality (1).

Theorem 1. *The upper and lower guaranteed values $\{\hat{V}_t\}_{t\in N_0}$ and $\{\check{V}_t\}_{t\in N_0}$ satisfy a.s. the recurrent relations, $t \in N_0$,*

$$\begin{cases} \hat{V}_t = \operatorname*{ess\,inf}_{\gamma\in U_{t+1}} \operatorname*{ess\,sup}_{\mathsf{Q}\in\mathfrak{R}} \mathsf{E}^{\mathsf{Q}}\left[\hat{V}_{t+1}e^{-\gamma\triangle S_{t+1}}|\mathcal{F}_t\right], \\ \hat{V}_N = e^{f_N}, \end{cases} \tag{2}$$

$$\left(\begin{cases} \check{V}_t = \operatorname*{ess\,sup}_{\mathsf{Q}\in\mathfrak{R}} \operatorname*{ess\,inf}_{\gamma\in U_{t+1}} \mathsf{E}^{\mathsf{Q}}\left[\check{V}_{t+1}e^{-\gamma\triangle S_{t+1}}|\mathcal{F}_t\right], \\ \check{V}_N = e^{f_N} \end{cases}\right). \tag{3}$$

To prove Theorem 1, we justified non-strict inequalities in both directions for the left and right parts of (2) and (3). The issue is to rearrange the essential extremum and conditional mathematical expectation. One can verify that the sets for which optimization is carried out are all of them directed upwards [1]. So, the monotone convergence theorem allows for rearranging.

Now, let us establish existence conditions for a seller's strategies $\hat{\gamma}$ $(\check{\gamma}) \in U$ such that for any $t \in N_1$, one has a.s.

$$\hat{V}_{t-1} = \left(\operatorname*{ess\,sup}_{\mathsf{Q}\in\mathfrak{R}} \mathsf{E}^{\mathsf{Q}}\left[\hat{V}_t e^{-\gamma_t\triangle S_t}|\mathcal{F}_{t-1}\right]\right)\Big|_{\gamma_t=\hat{\gamma}_t} \tag{4}$$

$$\left(\check{V}_{t-1} = \operatorname*{ess\,sup}_{\mathsf{Q}\in\mathfrak{R}} \mathsf{E}^{\mathsf{Q}}\left[\check{V}_t e^{-\check{\gamma}_t\triangle S_t}|\mathcal{F}_{t-1}\right]\right). \tag{5}$$

Let us denote $\hat{\Upsilon} := \{\gamma \in U$ satisfies (4) for any $t \in N_1\}$ and $\check{\Upsilon} := \{\gamma \in U$ satisfies (5) for any $t \in N_1\}$. By $\mathfrak{M}$ we denote the set of all $(\mathcal{F}_t)_{t\in N_0}$-martingale probability measures defined on $(\Omega, \mathcal{F})$ [1, Definition 5.14].

Theorem 2. *The following statements are equivalent:*

1) $\mathfrak{R} \cap \mathfrak{M} \neq \varnothing$*;*

2) $\hat{\Upsilon}$ $(\check{\Upsilon}) \neq \varnothing$*;*

3) there are $\hat{\gamma}$ $(\check{\gamma}) \in U$ and $(\mathcal{F}_t)_{t\in N_0}$-adapted a.s. non-decreasing non-negative stochastic sequens $\hat{C} := \{\hat{C}_t\}_{t\in N_0}$ $\left(\check{C} := \{\check{C}_t\}_{t\in N_0}\right)$ with $\hat{C}_0 = 0$ $\left(\check{C}_0 = 0\right)$ such that self-financing portfolio with consumption $(\hat{\pi}, \hat{C})$ $\left((\check{\pi}, \check{C})\right)$, where the number of risky assets is determined by a strategy $\hat{\gamma}$ $(\check{\gamma})$ and $\hat{X}_t := X_t^{(\hat{\pi},\hat{C})} = \ln \hat{V}_t$ $\left(\check{X}_t := X_t^{(\check{\pi},\check{C})} = \ln \check{V}_t\right)$, $t \in N_0$, has the following properties:

- *$f_N \leqslant \hat{X}_N$ $\left(f_N \leqslant \check{X}_N\right)$ a.s. and*
- *for any other portfolio with consumption (π, C): $f_N \leqslant X_N^{(\pi,C)}$ a.s. it follows that $\hat{X}_t \leqslant X_t^{(\pi,C)}$ $\left(\check{X}_t \leqslant X_t^{(\pi,C)}\right)$ a.s., $t \in N_0$.*

The essence of the proof is as follows. Implication from 1 to 2 follows from the fact that for any $t \in N_0$, random variables $\hat{\Phi}_t^{\gamma} := \operatorname*{ess\,sup}_{\mathsf{Q}\in\mathfrak{R}} \mathsf{E}^{\mathsf{Q}}\left[\hat{V}_t e^{-\gamma\triangle S_t}|\mathcal{F}_{t-1}\right]$ and $\check{\Phi}_t^{\mathsf{Q},\gamma} := \mathsf{E}^{\mathsf{Q}}\left[\check{V}_t e^{-\gamma\triangle S_t}|\mathcal{F}_{t-1}\right]$ are a.s. finite, convex, continuous functions of γ, and for any $\{\gamma_k\}_{k\geqslant 1}$ such that $\lim\limits_{k\to\infty}||\gamma_k||_{\mathbb{R}^d} = \infty$ a.s. the limit $\lim\limits_{k\to\infty}\hat{\Phi}_t^{\gamma} = \infty$ $\left(\lim\limits_{k\to\infty}\check{\Phi}_t^{\mathsf{Q},\gamma} = \infty,\ \mathsf{Q}\in\mathfrak{R}\right)$ a.s. One can prove implication from 2 to 3 directly by writing the corresponding inequalities for the portfolio with number of risky assets defined by $\hat{\gamma} \in \hat{\Upsilon}$ $\left(\check{\gamma} \in \check{\Upsilon}\right)$ and $\triangle\hat{C}_t^{\hat{\gamma}} := -\triangle\ln\hat{V}_t + \hat{\gamma}_t\triangle S_t$ $\left(\triangle\check{C}_t^{\check{\gamma}} := -\triangle\ln\check{V}_t + \check{\gamma}_t\triangle S_t\right)$, $\hat{C}_0 = \check{C}_0 = 0$, $t \in N_1$. Implication from 3 to 1 follows from the contradiction between item 3 of the theorem and the existence of $t \in N_1$ and non-zero $\gamma' \in U_t$ such that $\mathsf{P}(\gamma'\triangle S_t < 0|\mathcal{F}_{t-1}) = 0$ and $\mathsf{P}(\gamma'\triangle S_t > 0|\mathcal{F}_{t-1}) > 0$ a.s. It immediately proves that $\mathfrak{R}\cap\mathfrak{M} \neq \varnothing$ [10, Lemma 3, p. 422].

Remark 1. The proof of Theorem 2 implies that $\gamma \in U$ in items 2 and 3 is the same object. Thus, items 2 and 3 of the theorem suggest the way to construct a super-hedging portfolio with minimal initial value, which also allows us to calculate the value of such a portfolio at any time moment $t \in N_0$.

Remark 2. The values $\hat{X}_t$ и $\check{X}_t$, $t \in N_0$, are a.s. minimal, and $\hat{C}_0 = \check{C}_0 = 0$. So, item 3) of Theorem 2 implies that there is always an equilibrium in the game Γ with an arbitrage-free market model (i.e., model with $\mathfrak{R}\cap\mathfrak{M} \neq \varnothing$).

3. Examples

Suppose one-dimensional random variables $\{S_t\}_{t\in N_0}$ are defined by recurrent relations

$$S_t = S_{t-1}\left(1+\rho_t\right), \quad S_t|_{t=0} = x_0 > 0,$$

where $\{\rho_t\}_{t\in N_1}$ are i.i.d. with support $\{a_1, ..., a_l\}$, $-1 < a_1 < ...a_i < 0 < a_{i+1}... < a_l < \infty$, $1 \leqslant i < l$, and respective probabilities $p_j > 0$, $j = \overline{1,l}$, $\sum\limits_{j=1}^{l} p_j = 1$. Then $(\Omega, \mathcal{F}, \mathsf{P})$ is the corresponding probability space. Let $\mathcal{F}_t := \sigma(S_s, s \leqslant t)$, $t \in N_0$. Here S_t is used for the discounted value of a risky asset unit, and ρ_t is its yield, $t \in N_0$. It is known that this market model with $l > 2$ is incomplete.

For a European contingent claim $f_N = f(S_N)$, where $\mathsf{P}(f_N \geqslant 0) = 1$ and f is convex, solution for the minimax problem is well known [9], $t \in N_1$:

1) $\mathsf{Q}^*(\rho_t = a_1) = q^*$, $\mathsf{Q}^*(\rho_t = a_l) = 1 - q^*$ and $\mathsf{Q}^*(\rho_t = a_j) = 0$, $j \notin \{a_1, a_l\}$, where $q^* = a_l/(|a_1| + a_l)$;

2) $\ln V_t(x) = \sum\limits_{s=1}^{N-t} C_{N-t}^s f(x(1+a_1)^s(1+a_l)^{N-t-s})(q^*)^s(1-q^*)^{N-t-s}$;

3) $\gamma_t^* = \frac{1}{S_{t-1}(|a_1|+a_l)}\ln\frac{V_t(S_{t-1}(1+a_l))}{V_t(S_{t-1}(1+a_1))}$;

4) the number of risk-free asset units might be calculated via self-financing condition, and one may chose $\beta_0 = \ln V_0$.

There in Theorem 2, it was proved that such a portfolio is a super-hedging one with minimal initial value.

Let us apply game theory to calculate a vanilla European call for gold. We use the opening quotes for a troy ounce of gold for the period from 03 September 2007 to 25 November 2023, published at https://www.finam.ru/profile/tovary/gold/export/. From the data, it follows that prices for gold varied in the range from 674.90 to 2066.30 dollars for one troy ounce, and its yield varied from -0.100781 to 0.079597. So, we are able to price the European call option for gold with $f_N(x) = \max\{0, S_N - K\}$, where K is a constant. Suppose we will price it for a monthly contract, specifically for August 2023. We find: $S_0 = 1986.2$, $N = 26$, $a_1 = -0.100781$, $a_l = 0.079597$, $q^* = 0.441279$, $1 - q^* = 0.558721$. If $K = 1900$, than $\hat{X}_0 = 398.12$ dollars. A superhedging portfolio, its value, consumption, and its value minus consumption are in the Table.

Table. Pricing results for month European call option for troy ounce of gold

Date	Price for troy ounce of gold	Number of risky assets units	Number of risk-free assets units	Value of portfolio	Portfolio value minus consumption	Consumption
01.08.23	1 986.20	0.000000	398.12	398.12	398.12	0.00
02.08.23	1 972.60	0.619681	−832.69	389.69	385.76	3.93
03.08.23	1 970.30	0.613819	−821.13	388.28	376.65	11.64
04.08.23	1 977.10	0.610657	−814.89	392.43	372.19	20.25
06.08.23	1 974.00	0.613040	−819.61	390.53	366.59	23.95
07.08.23	1 967.80	0.610268	−814.13	386.75	348.44	38.31
08.08.23	1 963.90	0.605305	−804.37	384.39	345.80	38.59
09.08.23	1 951.20	0.603263	−800.36	376.73	325.52	51.21
10.08.23	1 946.60	0.594048	−782.38	374.00	319.03	54.96
11.08.23	1 945.50	0.591792	−777.99	373.35	309.19	64.16
13.08.23	1 944.60	0.588073	−770.75	372.82	298.83	73.98
14.08.23	1 939.40	0.587151	−768.96	369.76	290.57	79.19
15.08.23	1 934.90	0.581689	−758.37	367.15	271.55	95.60
16.08.23	1 922.30	0.576632	−748.58	359.88	263.09	96.79
17.08.23	1 924.10	0.567866	−731.73	360.90	248.12	112.79
18.08.23	1 917.70	0.564413	−725.08	357.29	240.22	117.07
20.08.23	1 920.40	0.560673	−717.91	358.80	229.68	129.12
21.08.23	1 924.70	0.556741	−710.36	361.20	218.99	142.20
22.08.23	1 929.90	0.560426	−717.45	364.11	215.83	148.28
23.08.23	1 948.90	0.559876	−716.39	374.75	205.40	169.35
24.08.23	1 941.00	0.572266	−740.54	370.23	196.86	173.37
25.08.23	1 941.40	0.567633	−731.55	370.46	178.53	191.93
27.08.23	1 944.10	0.559143	−715.06	371.96	164.48	207.48
28.08.23	1 953.30	0.572692	−741.40	377.23	158.12	219.11
29.08.23	1 964.70	0.569838	−735.83	383.73	125.33	258.40
30.08.23	1 970.40	0.612307	−819.27	387.22	126.96	260.26
31.08.23	1 965.50	0.639355	−872.57	384.09	65.50	318.59

Conclusion

We reformulated the problem of European option pricing on an incomplete market with discrete time and a finite number of assets, without transaction costs and trading restrictions as the dynamic stochastic antagonistic game Γ of the market and the option seller. It was shown that the upper and lower guaranteed values satisfy a.s. the Bellman-type recurrent relations. So, the implementation of the stochastic dynamic programming method is justified. As it turns out, the game equilibrium and self-financing portfolios, at which the minimum in the recurrent relations is reached, exist if and only if the market model is an arbitrage-free one. These portfolios are superhedging, and their value is minimal for a superhedging portfolio. So, the game suggests how to construct a superhedging portfolio with minimal value for a European option. Analytical and numerical examples are given.

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Поступила в редакцию / Received 23.11.2023
Принята к публикации / Accepted 12.12.2023
Опубликована / Published 02.03.2026